Working with the left side of the diagram is called the Hamiltonian, which gives us the stress strain curve.
Working from the right side of the diagram is called the Lagrangian, giving us the strain energy curve.

The refractive plate on the right side of the diagram, has been placed such that the loading on the truss follows the frequency corresponding to the linear Euclidean scale. By putting this plate we have turned the curve on the Lagrangian (right) side is a straight line. Next we generate the velocity pole curve, which will act as our projection plane or ellipse of projection, where they will refract and meet at the north pole of C4 we can when passing through the hyperplane, the horizontal axis. If the rays pass straight through, they will meet and create the stress strain curve. We can also create the eliptic curve by assuming the vertical axis has bent, and rotated, but we do not know the extent of this rotation.

It is important to note that the material, in this case steel, has an inherent disturbance potential at the resonant frequency of 80 x 60 / 360 = 13.33 Hz. Load /stress reversal due to wind or earthquake loads at this frequency shall be at magnitudes below the Fr (point 5) on the stress strain curve.

T= 1/13.3 = 0.075 a very good number for the period.

Take a unit circle with a radius of 1. the circle is divided into 60 seconds so that it goes one cycle in 60 seconds. Also 360° degrees during that same revolution. To convert :
80 sec / cycle x 1 cycle / sec x 60 sec / cycle : 360° / cycle
80 sec / cycle x 60 = 360° / cycle hence the period T in seconds = 360 / 80 x 60

For a cantilever member this frequency will be one half the frequency at roughly 6.6 HZ, with T = 0.15 seconds, as we take $\frac{1}{2}$ the circles. (The upper half above the segment).

Regardless of the structure, the load reversal at this frequency for every component shall be avoided at all cost. Though a high-rise may have a lower frequency (higher period) if the entire structure is considered, members of the first few floors will always be more rigid or stiffer by design with a higher frequency (lower period). It is then incorrect to design an entire tall building for forces plastic region.

Take a column and design it for twice the yield or resonance load. Then looking at the right side of the diagram diagram, we see that if it is stressed to twice the yield we are at point 5, on the energy diagram we drop to about point 2 and its allowable strength drops to below the yield. If damping is considered in the design, then this point 2 might be taken higher.

The energy dissipating mechanism placed in the frame should be designed so that it bypasses the 13.3 HZ frequency.

For gasses this reversal happens at 4 °or 8° limits, depending on the assumed end conditions as simply supported or cantilever, and working backwards towards the singularity.

Density of steel, $\rho$ = 0.284 lb/in³ = 490 lb / ft³ (roughly 8 times that of water) - Iron - Body centered cubic crystal structure.
Modulus of elasticity of steel E = 30 x 10^ 6 lb/in²

Recall that in a central force field the radius sweeps out area at a constant rate A called the areal speed. If the mass of the particle p is m, then the angular momentum L (= r x m v , I . $\omega$ ) of p is = 2 m A . The factor 2 comes from the fact that we reduce the area of the parallelogram from the addition of vectors to that of the triangle almost equal to the arc of the circle, and also that there are actually two arcs or wheels, which either cross or do not. Put another way, if we are considering a beam, then there are two reactions, one on either end, giving rise to two tangent velocity vectors.

Arc length = 444' $\rho$ = 0.248 lb(force)/in³
W = mg = 0.248 x 1 in² x 444 x 12 = 1321 lb(force) therefore m = 1321 lb / 32.2 ft/sec² x 12 in/ft = 3.4 lbs (mass)
Area of the arc = 2,147,363 in²
2 mA = 2 x 3.4 x 2147363 = 14682538.9 for two arcs = 29,365,077 lb in²
The unit for L is lb . in² this angular momentum will tend to rotate an area of 1 in². Assume a square area, then its moment of inertia has a unit of in^4 . So pick for E a unit of lb . in² / in^4 = lb /in².

An easier method take the area of segment of arc :

Take arc length of the circle x (12) x radius (12) = 444 x 12 x 475 x12 = 30.369 x 10^6 in²

Using this diagram, assuming the material has a similar packing density, one can rotate the plate of refraction and obtain the curve for any material. Note that if the structure changes to one which is denser say Hexagonal close packed (Titanium for example Commercial (99.2% pure) of titanium have of about 63,000 psi ), then the density will have to be adjusted, roughly $\frac{1}{2}$. Using the density of the material, one can also scale up or down the values as compared with the density of steel (BCC) with $\rho$ = 490 lb / ft³ .

For example, a material with lower density such as Aluminum (Face Centered Cubic), with a similar packing density, $\rho$ = 167 lb / ft³ , we would obtain a modulus of roughly $\frac{1}{3}$ that of steel, or 10.35 x 10^6 psi. Its resonant frequency will be 13.3 / 3 = 4.4 HZ if the refractive plate does not rotate. If the plate rotates its frequency will be 7.2 HZ. For a cantilever structure the the frequency will be 3.6 HZ and the period will be 0.27 sec.

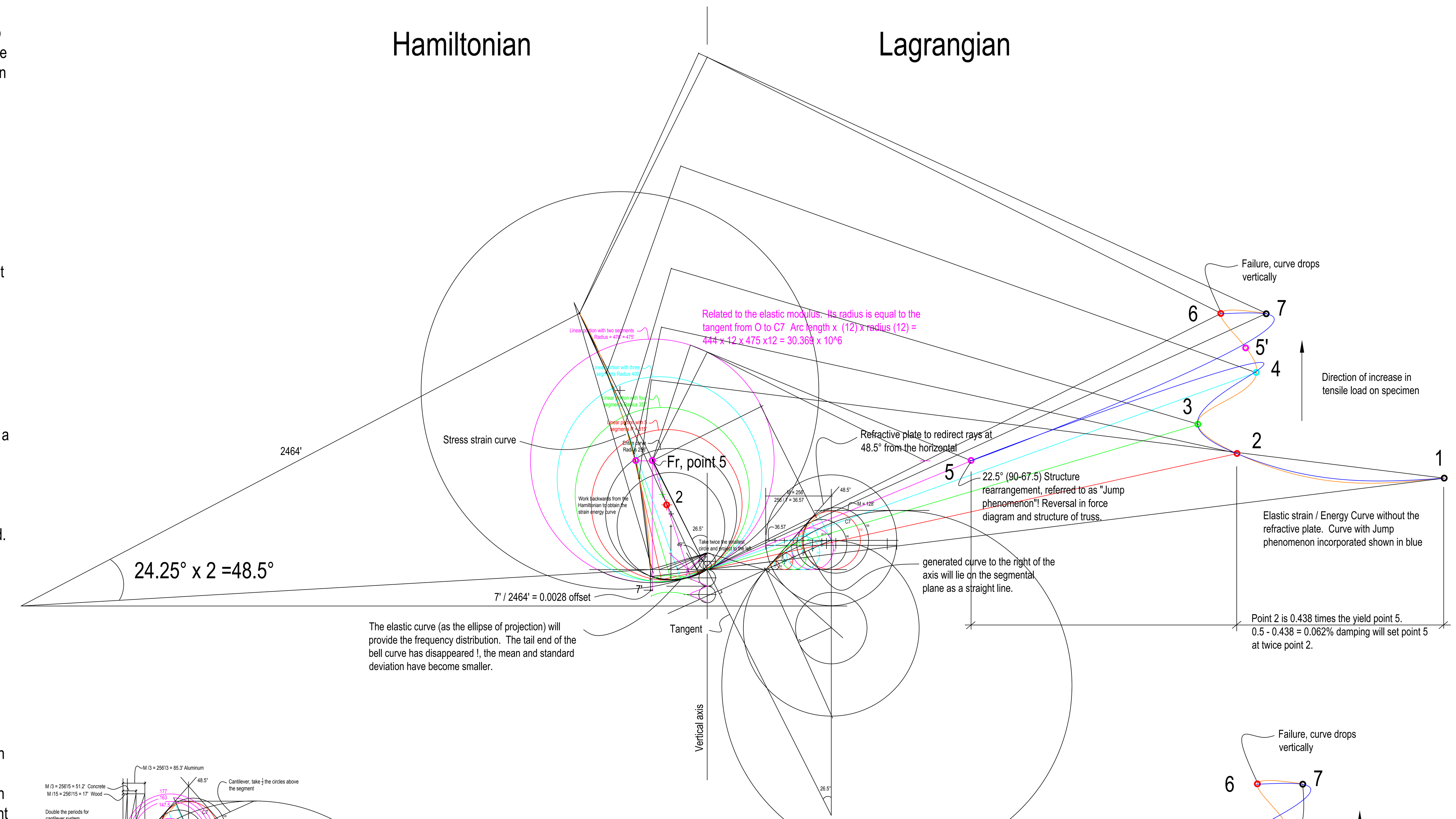
For wood, which has a specific gravity of 0.5, taking one half the density of water which is $\rho$ water = 62.4 lb / ft³ , $\rho$ wood = 31.2 lb / ft³ . The modulus decreases by 490 / 31.2 = 15.7 fold. E wood = 30.369 x E6 in² / 15.7 = 1.9 x E6 psi

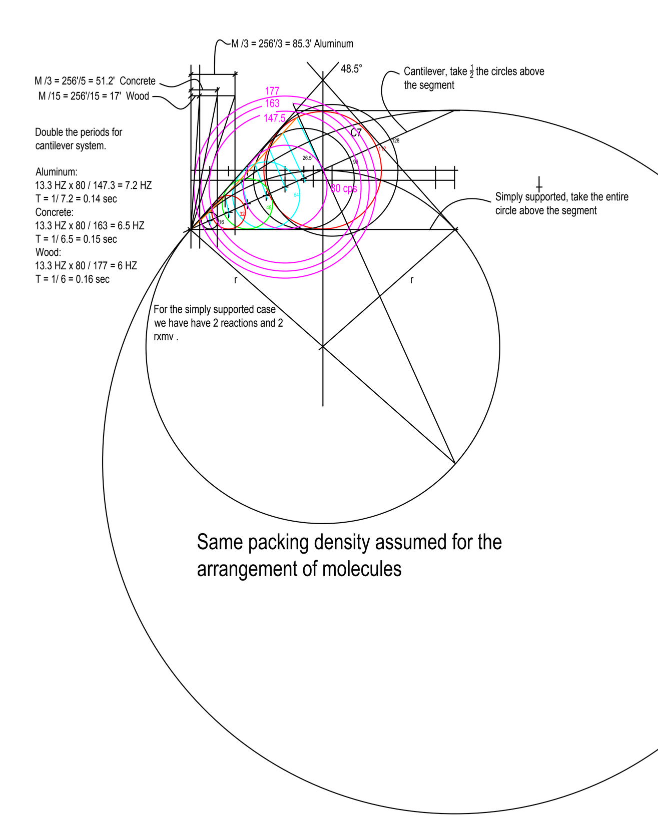
For concrete take the density of the three materials, sand, cement and gravel combined average: sand = 80 lb/ft³, cement = 85 lb/ft³, gravel = 85 lb/ft³ . total: 85. Take a water to cement ratio of 0.5 (As water will be leaving the material this factor is equal to (1- W/C)) this will take us to 42.5 lb/ft³, say 50 lb/ft³, or $\frac{1}{10}$ th the modulus of elasticity of steel, as 3.369 x 10^6 in².
Also its compressive strength would be roughly 10th of that of steel, about 5000 psi (taken as 3000 to 4000 psi)
The tensile strength given its porosity of 0.1 would be one tenth of its compressive strength = 400 to 500 psi.
For the frequency of concrete: 13.3 HZ x 80 / 163 = 6.5 HZ
T = 1/ 6.5 = 0.15 sec

For earth, we should take the void ratio into account as opposed to the packing density: range T= 0.1 to 0.2 seconds if confined.

Note that for the case where light is not absorbed by the material, we have the 4° limit. Combine the two energy diagrams, one of black hole singularity with this diagram.

# Stress strain - Strain energy curve



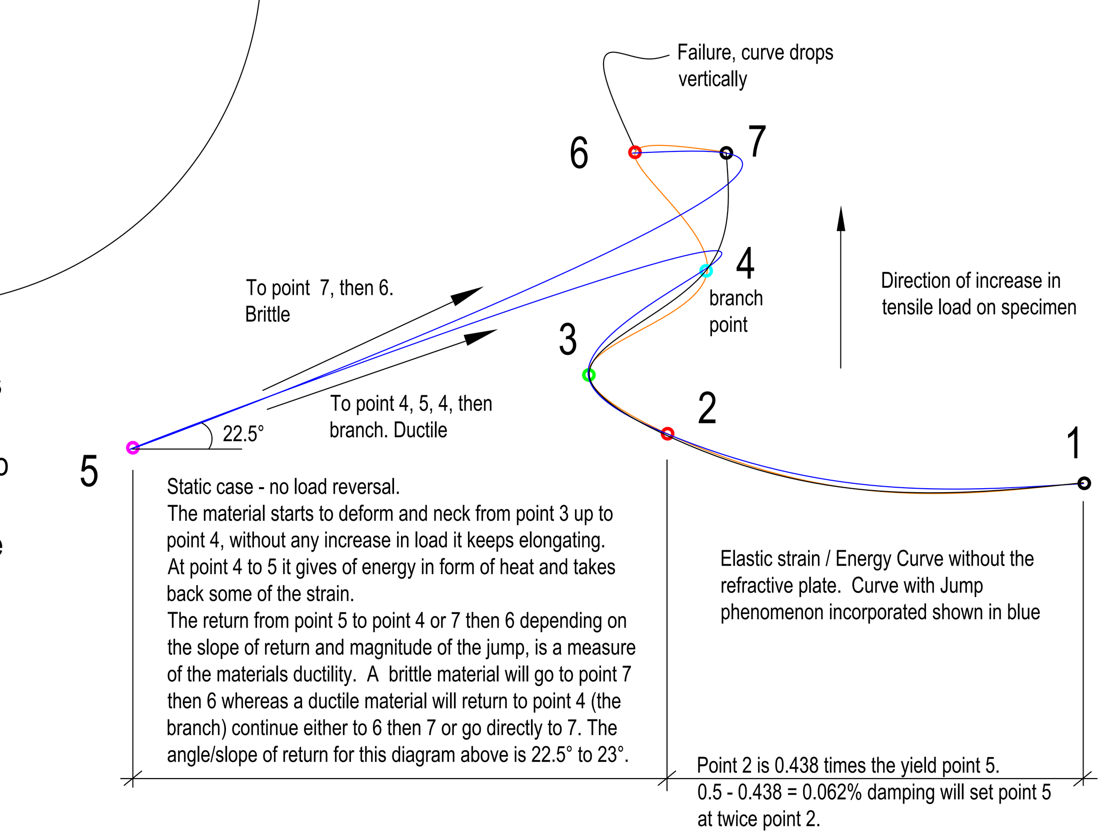




This damping is not constant and varries for nonlinear load deformation. To take the force / stress flow analogy through a building, Imagine a bucket of water with two holes underneath, one whose flow will equal to half the yield of the material and the other for 6% of that. If the holes are static and do not change, then water will flor in proportion to the size of the holes, but if the larger hole is enlarged as the our building deforms, then the water will flow through the larger hole and will stop leaving through the smaller hole if the rate of deformation is larger than 5%.

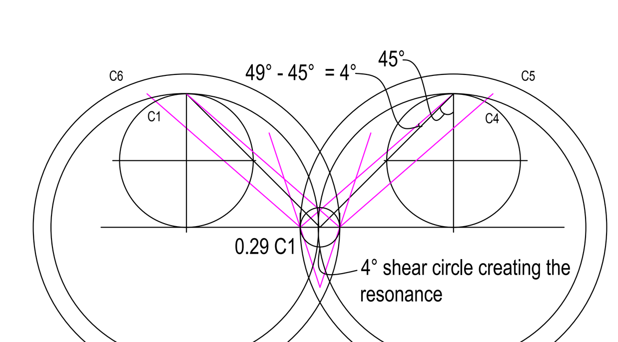
Note in going from point 4 to point 7 we have two options. One path is to go directly to point 6 or 7 if the resonance frequency is bypassed. If the resonance frequency is not bypassed, which could happen during load reversal, we would then jump to point 5, in which case we either return to 4 then branch out to 6 or go directly to 7 and then 6.

A brittle material from the jump point 5 will go to point 7, then 6, whereas a ductile material will go from 5 back to 4 then branch out.

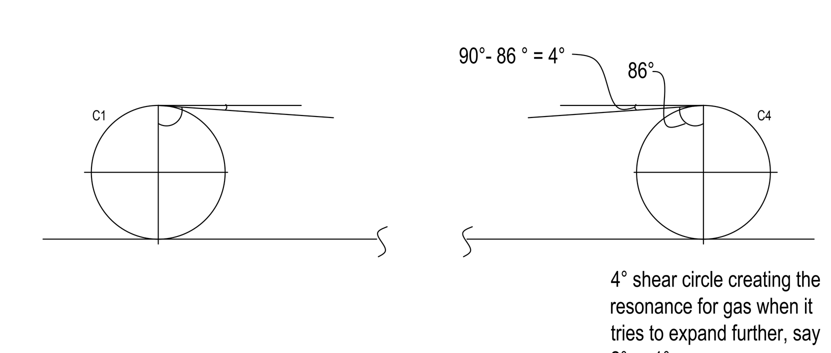
If we load the material from 1, 2, ... to point 5 and remove the load, it will return from 5 directly to point 1. The load can be removed gradually to point 4 then 2 then 1.




Strain energy curve for solids energy increases from point 1 going to the left

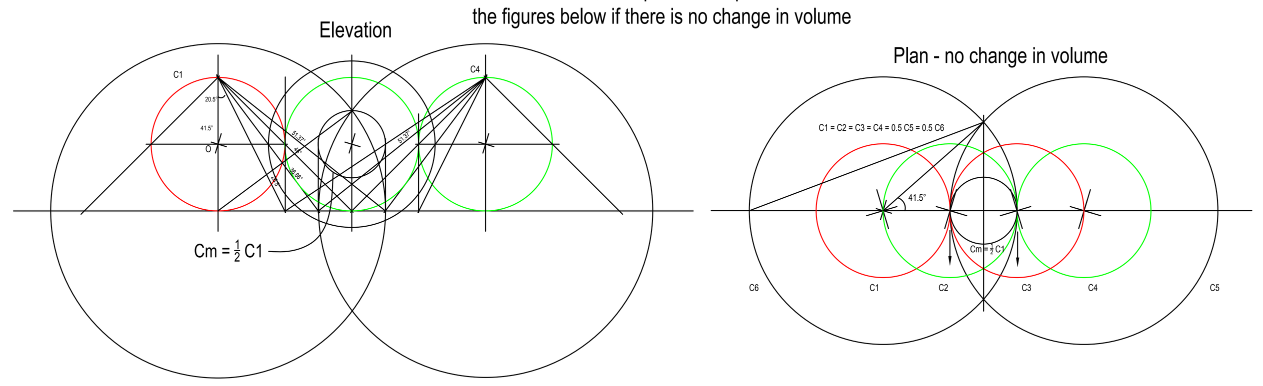



Enlarged (4x) for clarity - for material the shear circle will tend to hold the material together when stretched until Cm = $\frac{1}{2}$ C1 or $\frac{1}{2}$ C4 at 51.37°. C5, and C6 will expand and contract for constant volume

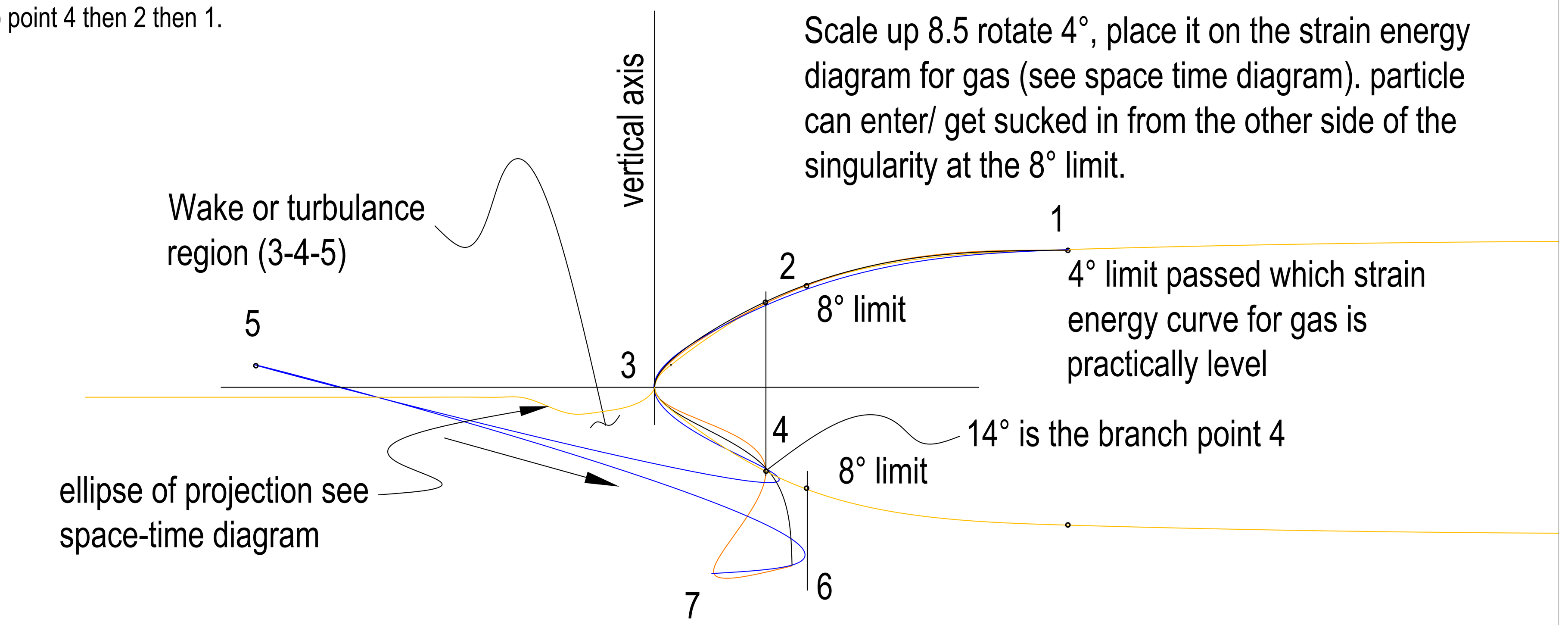



Same diagram for gas - circles are infinite distance apart, the shear circle will compress and reduce the curvature of the space

For the material, the space will expand and contract as the figures below if there is no change in volume




Scale up 8.5 rotate 4°, place it on the strain energy diagram for gas (see space time diagram). particle can enter/ get sucked in from the other side of the singularity at the 8° limit.




For gas, invert the diagram scale it up 8.5 fold and place point 1 on the 4° limit. Think in terms of turbulence or wake generated as we move the singularity, point 3, back and forth with the strain energy curve as our body moving through a fluid. In the case of a wake, as the singularity moves to the right, the particle will get sucked in and in the case that the singularity moves to the left, there is a turbulence region where the particle can not enter and will go from 5 to 6, then 7, and leave. The particle can however enter from the side, at point 6, at 90 degrees to the horizontal axis but not if traveling parallel unless the singularity moves to the right and we have a wake region created behind the singularity. A bridge can theoretically connect point 5 occurring on both side of the singularity.