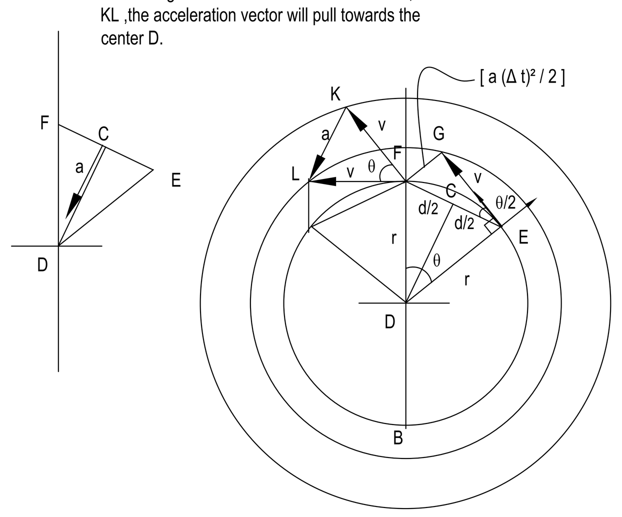
Potential Energy Function, Hypercubes and Light Cones

Travel from E to F with velocity v.
At F the acceleration will be equal to KL = a.
As the angle θ becomes smaller and smaller, KL ,the acceleration vector will pull towards the center D.



Let the particle travel from E to F. From E draw the tangent velocity and from F drop a perpendicular, FG, to this velocity vector v.

FG, the distance the particle falls towards D, will then be equal to $1/2\, a\, (\Delta t)^2$

As the angle θ becomes smaller ΔS = d , ΔS /2 = d /2 . Using similar triangles DCF and EGF we can write:
$(d/2) / r = [a (\Delta t)^2 / 2] / d$
$d / r = a (\Delta t)^2 / d$
$(d / \Delta t)^2 / r = a = v^2 / r$

Assume EG is parallel to FK
Triangles DEF and FKL are similar
As the angle θ becomes smaller, we may replace arc EF by the distance d (= ΔS).
Hence: d / r = a / v

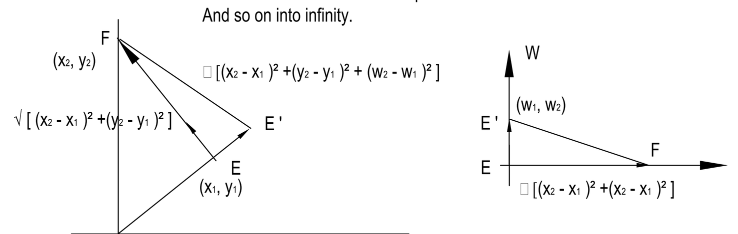
The distance d may be covered in t seconds, and hence d = v t , and a = b t,

$b t / v = v t / r$ or $b = v^2 / r$

In the figure above, working inwards in two dimensional space, taking EG as the square root of two numbers, then taking the Normal to EG will give us a third dimension in two dimensional space.

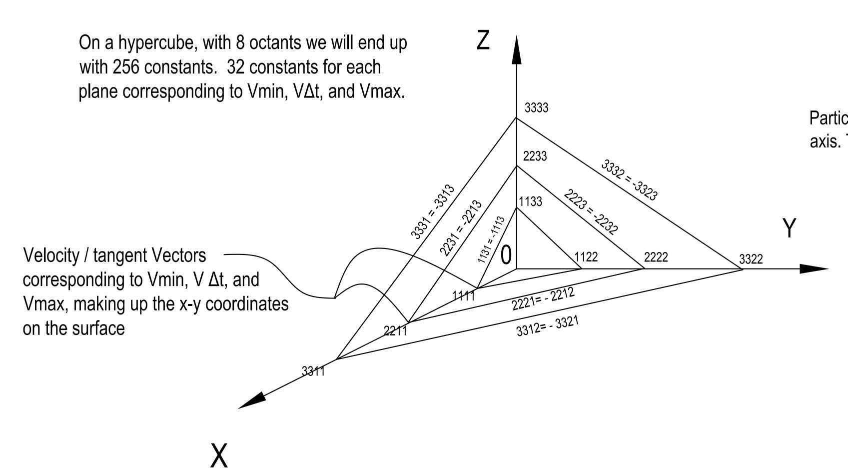
In the figure to the left we go outwards from EF, with E E' making the diagonal E'F.

The new hypoteneus, E ' F, will be the third dimension in two dimensional space.
And so on into infinity.



In three dimensional space
The distance for the line is equal to:
$\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
Take a normal in any direction and the diagonal discance then becomes:
$\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 + (w_2 - w_1)^2]}$
This is the four dimensional space in the space of three dimensions, and so on,... now the curve could be plane or planar. If plane then it is in two dimensions and if planar in three dimensions.

On a hypercube, with 8 octants we will end up with 256 constants. 32 constants for each plane corresponding to Vmin, VΔt, and Vmax.



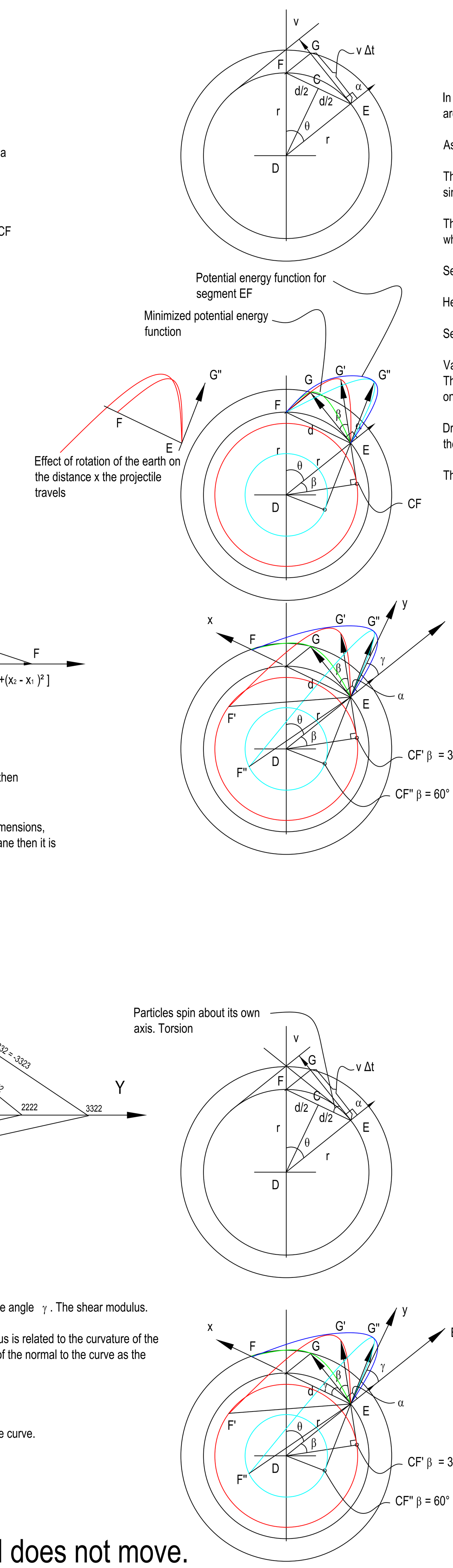
The angle gama γ is related to the spin of the particle about its own axis.
If it is desired to include this in finding the distance x the projectile will travle then use axis EG .
To neglect the effect of the particles rotation about its own axis we can rotate the x-y axis by the angle γ . The shear modulus.

So then the modulus of rigidity is related to the change in elevation angle and the shear modulus is related to the curvature of the curve and if the curve can be approximated by a circular function, then by taking the direction of the normal to the curve as the reference point the function becomes analytical at that point.

How do we then minimize the potential energy function? and what does this mean?

We would differentiate the function and set it equal to zero. We can then find the tangent to the curve.

Starting in reverse from a point, we would have to start with rotating γ .



In order to see the connection between the sector velocity and the angular momentum, calculate the area wwept over by the radius from the central point D to the particle in a small time Δt .

As the angle θ becomes smaller v = d (= ΔS)

The triangular area swept over by the radius is = [v . Δt x r sin □] / 2 for the circular orbit, the sin of 90° = 1 and so the area swept = v . Δt x r /2

The average sector velocity during this time Δt is then = v r sin □ /2 , or expressed as $v_n . r/2$ where $v_n = v \sin \alpha$ where $v_n = r . □$, ω being the angular velocity, we have:

Sector velocity = $r . v_n / 2 = 1/2\, r^2 \omega$

Hence we see that the sector velocity is proportional to the angular momentum $L = m\, r^2\, \omega$

Sector velocity = L / 2m

Vary the angle of projectile by and angle β . The new center of force will be at an angle β from DE. The path of the projectile was from E to G to F, a cubic curve and now has changed to a parabolic one.

Draw the envelope by varying β (the elevation angle), shown in blue, in the case that the rotation of the earth, with a uniform acceleration in the clockwise direction, has been taken into account.

This envelope will provide us with the potential energy function.

To determine the path of the projectile, neglecting the clockwise rotation of the earth, while we fire the projectile at different angles. Find the new center of force and the corresponding sphere as the angle β is increased. The earth would get smaller and the projectile would land on this smaller sphere. Finally draw the envelope shown in blue.

In the figure to the left, as the angle β is increased to 30, 60, etc., dx = 0, dy/dx = ∞ and the circle at D becomes smaller and smaller, eventually shrinking to a point.

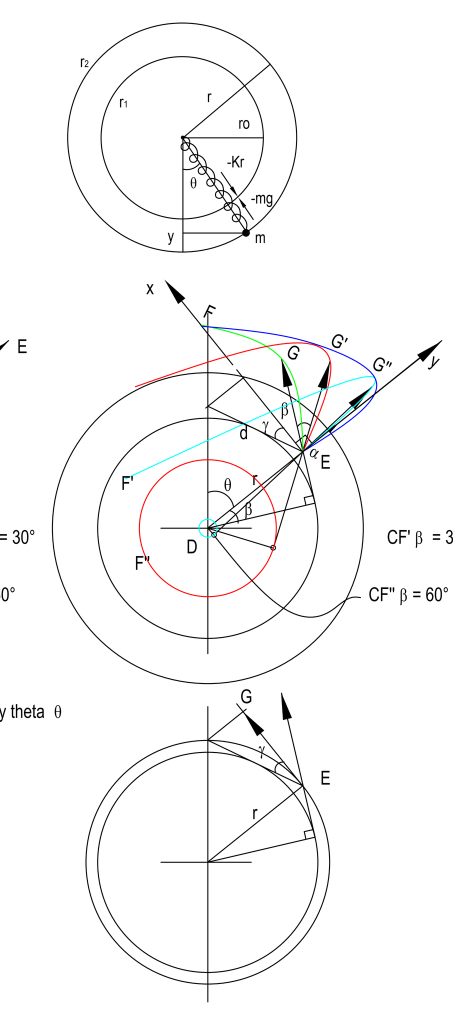
The motion at G'' at an angle β = 60 is hence more rigid than that at an angle 30° corresponding to G'.

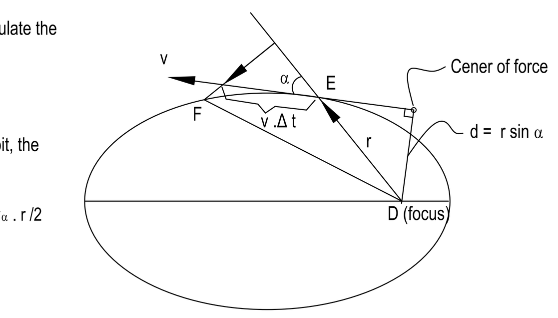
Furthermore, the figure to the left includes the rotation of the particle about its own axis. To factor out this rotation we have to rotate the x-y axis so that the y axis lines up with the radius DE

It should be pointed out that the motion of the projectile is plane (there is no motion in the z direction). The effect of out of plane motion of the projectile can be taken into account by vector addition, hence many possible paths eventually reduce to three for any path or curve.

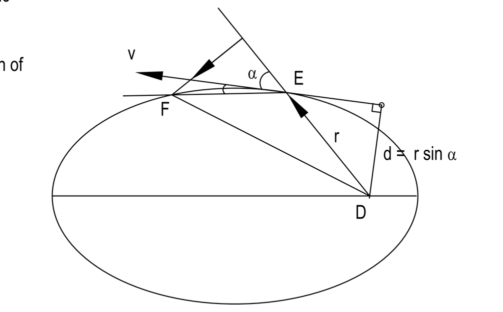
On a surface, we would have two velocities each with its minimum maximum and intermediate value leading to six differential equations. The velocity vectors on this surface when crossed will provide with the flow through the surface, and the vector resulting from this cross product gives us the third, z, dimension.

Inverse square force field:
E = Kinetic Energy , V = Potential Energy
K = Spring Constant
x = r2 - r1
potential energy $V = 1/2\, K\, x^2$
this is how we relate the potential energy of the projectile. As the angle of the projectile is increased the sphere decreases in size and the spring is hence stretched.

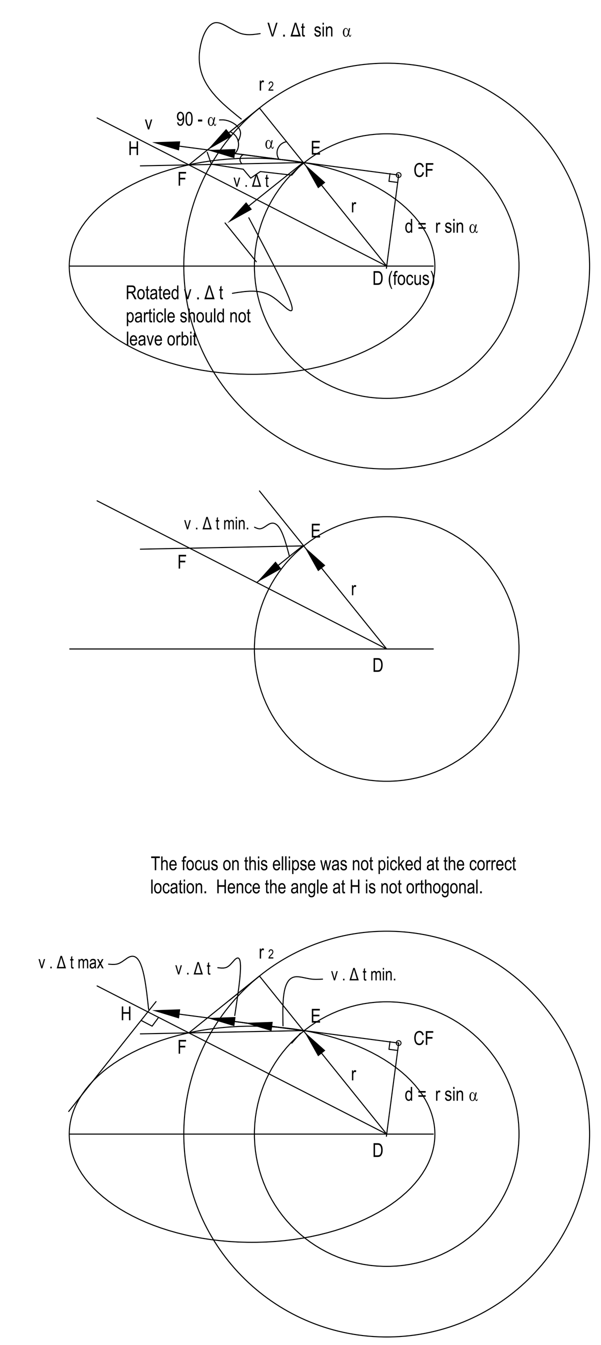




The area of the triangular sector DEF
A = d . FE = v Δt . r sin α
As the path becomes circular, the center of force and center of curvature conincide.



The focus on this ellipse was not picked at the correct location. Hence the angle at H is not orthogonal.



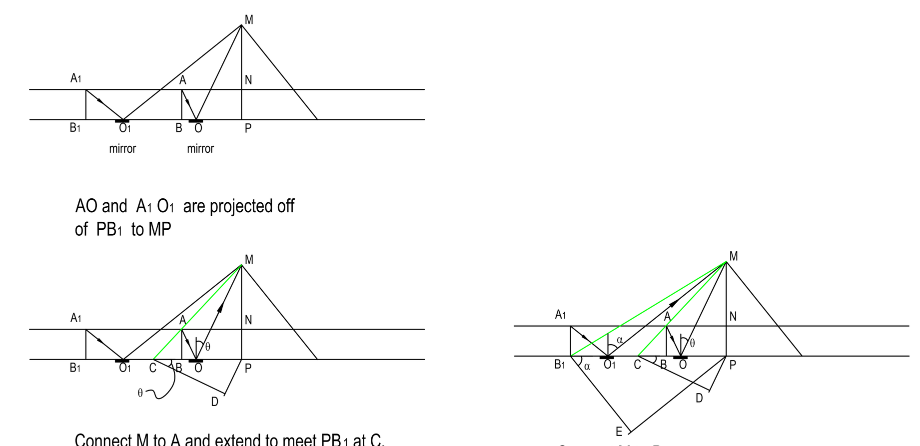
Side note - projection and antiprojection

Projection and Anti projection

The height of MP is as many times greater than the observer AB as the distance of the mirror from MP is greater than the distance of the observer from the mirror.
MP/AB = OP/OB

Now, suppose MP is inaccessible.
Place the mirror at O
MP/AB = OP/OB
OP = OB . MP / AB
Next, place the mirror at O1 and for this position we have
MP / A1B1 = O1P/O1B1
From this proportion we have:
O1P = O1B1 . MP / A1B1 Remember A1B1 = AB
Subtract OP from O1P. We have then
O1P -OP = O1O = O1B1 . MP / AB - OB . MP / AB
or
O1O = MP / AB . (O1B1 - OB)
and finally:
MP = AB . O1O/ (O1B1 - OB)

The height of the object is equal to the product of the height of the observers eyes from the grount by the ratio of the distance between the mirror positions to the difference between the distances of the mirror from the observer.



AO and $A_1 O_1$ are projected off of PB_1 to MP

Connect M to A and extend to meet PB_1 at C.
The angle AOM = 2 θ
From point C draw a line with an angle θ to the line PC
From point P draw a line parallel to MO
This line will intersec the antiprojection of CP at D
CD is then the antiprojection of CP on OM.
Similarly for A1O1

Connect M to B_1
The angle $A_1 O_1 M$ = 2 α
From point B_1 draw a line with an angle α from the line PB_1
From point P draw a line parallel to $O_1 M$
This line will intersec the antiprojection of PB_1 at E.
$B_1 E$ is then the antiprojection of PB1 on MO_1

We have two spheres. One with radius r and one where the radius increases.
V . Δt is tangent to the ellipse. In this case as we see the velocity is so large that the particle is about to exit the orbit as when we rotate v Δt we have passed the axis DF.

V . Δt sin α is tangent to the larger circle.

r_2 is the radius of curvature for the segment and to find the center of curvature we have to rotate r_2 by 90° - α

What length of segment should we choose for the given elliptical path and what will be the length of the velocity vector tangent to the ellipse? The smaller the segment and area DEF, the closer our approximation will be by using the area of the triangle DEH with the area equal to the base EH times the height d = D-CF.

Using iteration we can find the length of the velocity vector V. Δt min. as the tangent to the circle r_1 at E, to where it intersects DF. This length would then provide us with the length of the segment we should choose.

Path of travel of the Center of curvature for an elliptical path of travel is cubic in nature and r_F should equal r_2 .

Tha particle shall stay the course chosen.

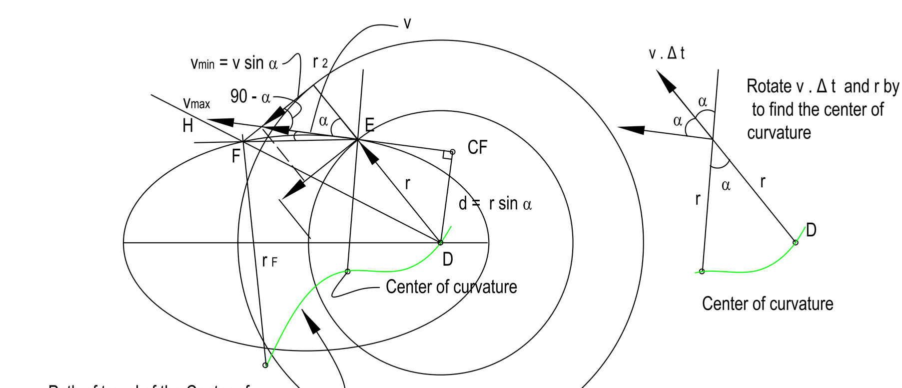
If and when the particle leaves the elliptic orbit, the process becomes unstable.

If the velocity is taken as less than v min = v sin α_E the process is unstable.

It becomes obvious that the solution to the cubic equation is determined if the length of the segment EF, the radius r_E, the angle α_E and the angle α_F are known, providing us with points D, D_m and D_F on the curve.

If traveling on a straight path in the direction of EF , we can continue this process for curves of higher order.

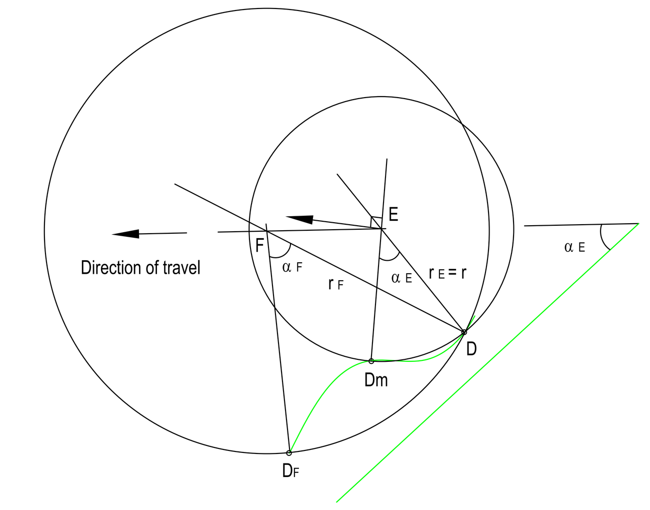
In two dimensions, on a surface, we would have two velocities each with its minimum maximum and intermediate value leading to six differential equations. The velocity vectors in two dimensions on this surface when crossed will provide with the flow through the surface, and the vector resulting from this cross product gives us the third, z, dimension.



Path of travel of the Center of curvature for an elliptical path of travel is cubic in nature and r_F should equal r_2 .

r_F should equal r_2 but as the angle velocity fluctuates so does the angle α .
To relate it to the velocity pole curve, and the tractrix, take $\alpha_E = \alpha_F = \alpha$, the angle of friction.
Then the tangent to all the circles becomes a maximum, perpendicular to the radii at the point of contact .
The cubic curve then becomes the deviation from the straight light cone with an angle α_E .

From one circle to the next, this light cone has then fluctuated. Recall we represented this with the lense.



Rotation of the light cone:

In terms of Decartes diagram, we are travelling on path or segment D to G (instead of EF) and our light cone has deviated to CS .

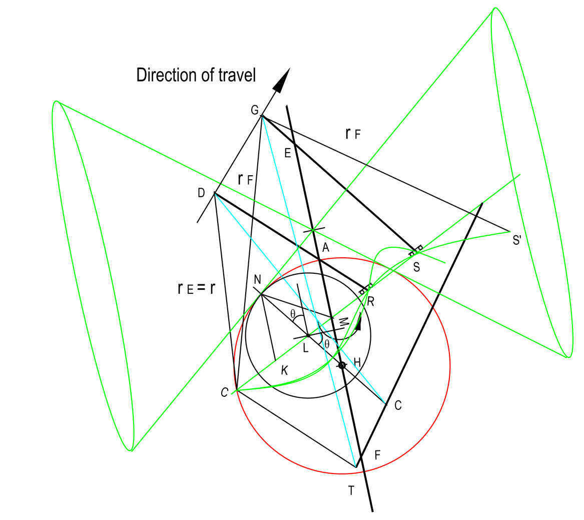
Note that GS should equal r_F

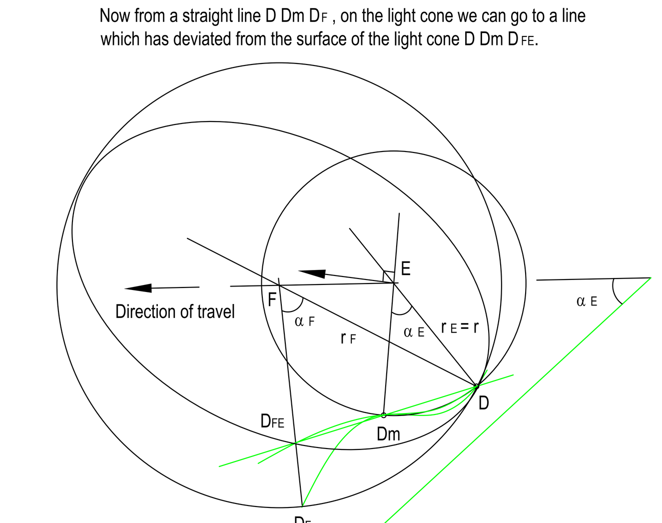
The light cone when rotated results in a change in r_F from GS' to GS. The circle at G becomes an ellipse. We will need to resort to elliptical functions, or rotate our light cone and solve it using circular functions..

Points C, R, and S would be in one line whereas C, R, and S' are not.

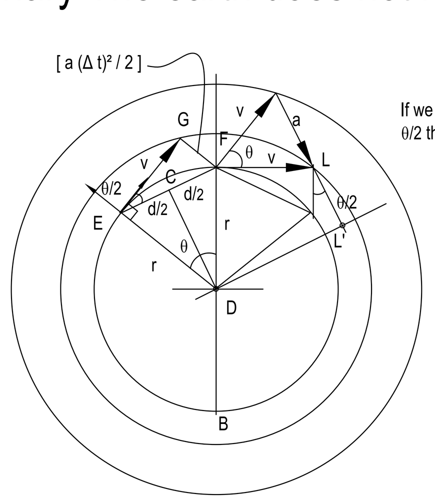
Draw a straight line from D to Dm, and extend to r_F . Find new D_F, call it D_{FE} We have minimized the deviation from the light cone as it is rotated. Using an elipse with its major axis along the radius r_F.

Now from a straight line D Dm D_F , on the light cone we can go to a line which has deviated from the surface of the light cone D Dm D_{FE}.

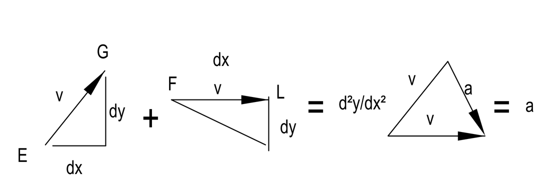


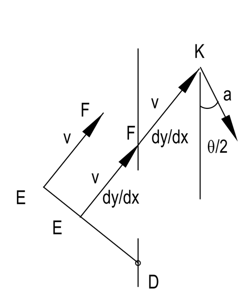


Irrotational - and Stationary The earth does not rotate and does not move.



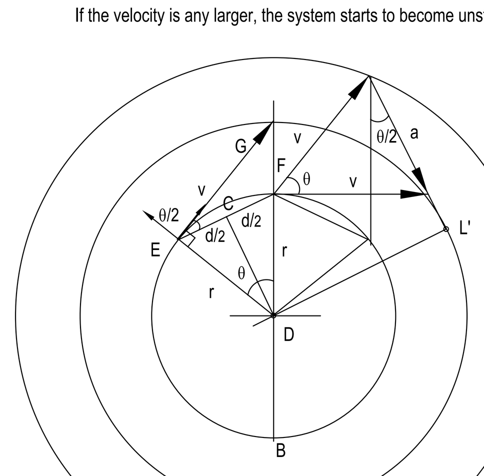
The rotation of the circle from E to F by angle θ which amounts to translation of the circle from D to L for there to be zero moment about point D.
Plus the rotation of the particle about its own axis by θ/2 for two dimensions = 1 /2 dy/dx



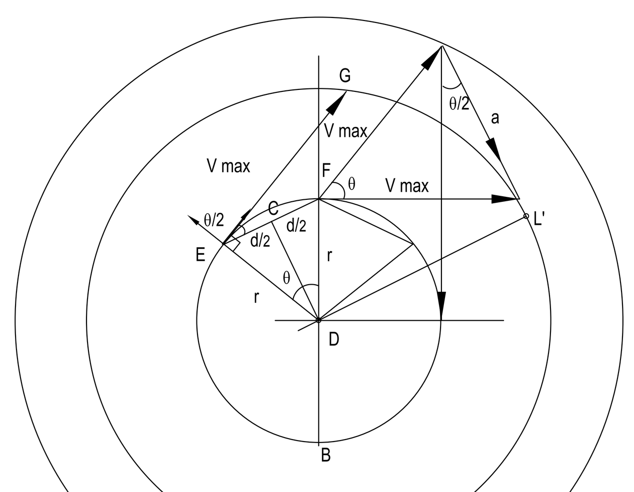


If we assume the earth rotates in a clockwise direction with angle θ/2 then the system becomes unstable at this velocity.

If the velocity is any larger, the system starts to become unstable



If the velocity is any larger, the particle will leave the orbit and there is no solution.



The Earth, now call it the object, has started to move and as it does it gains mass.
It will move in a straight line unless acted upon by an external force.

Next assume this force acting in a direction perpendicular to the direction of motion.

Based on the magnitude of the force we can draw the circular orbit of the object.

As an alternate, if we know the rotation of the earth, we can also determine this force which is perpendicular to the direction of motion.

Extend vector G to meet at a point which will be the center of the orbit.

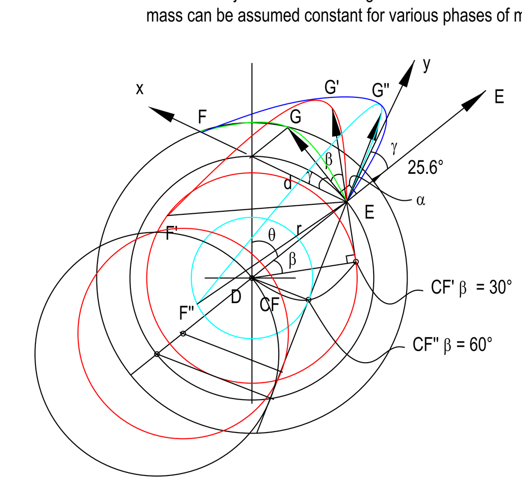
The rotation of the object should equal γ . And having determined the radius of curvature R, we can then say that if a force larger than that is applied to the object the object it will overcome the objects interia and the object will move.

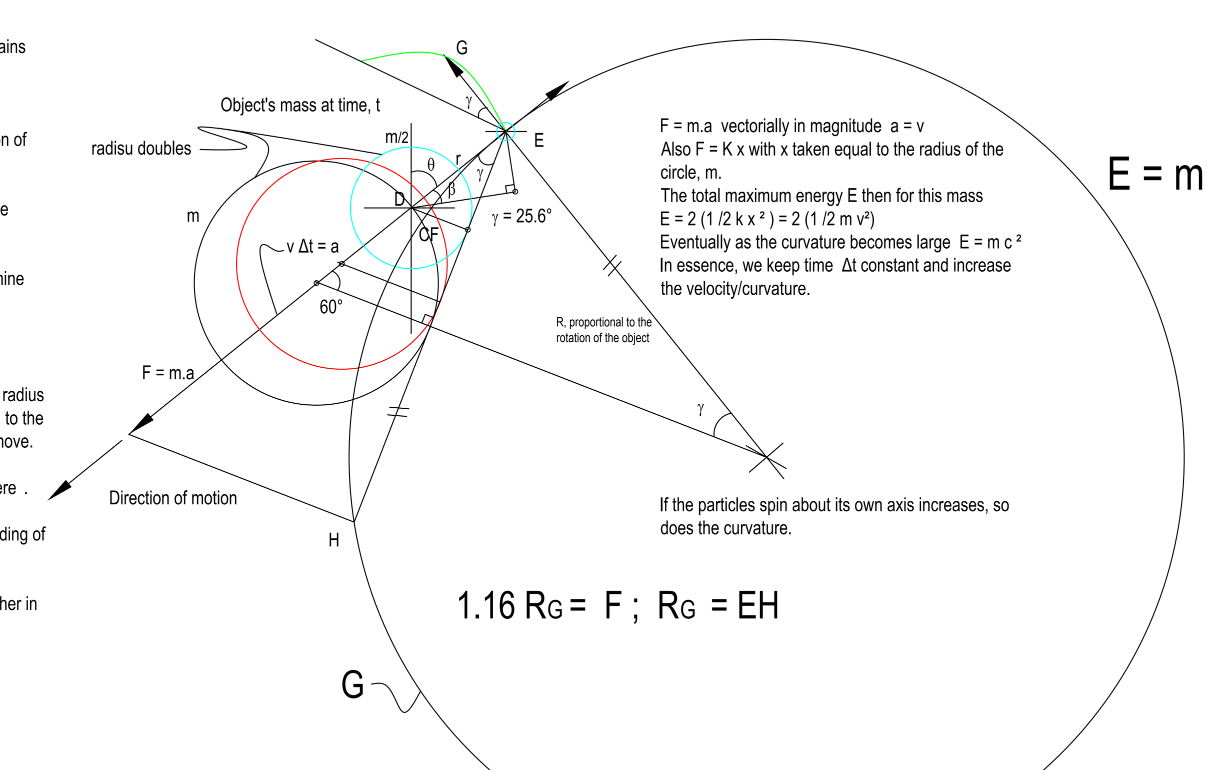
Next project the circles of mass on the circle and eventually on the sphere .

Rotation of the mass about its center of gravity will be equivalent to bending of the bottom chord of the Pratt truss into a Scissors truss.

For equilibrium, we should have one projectile in one direction and another in the opposite direction.

Move the Object /Earth assuming it has avariable mass. The mass can be assumed constant for various phases of material





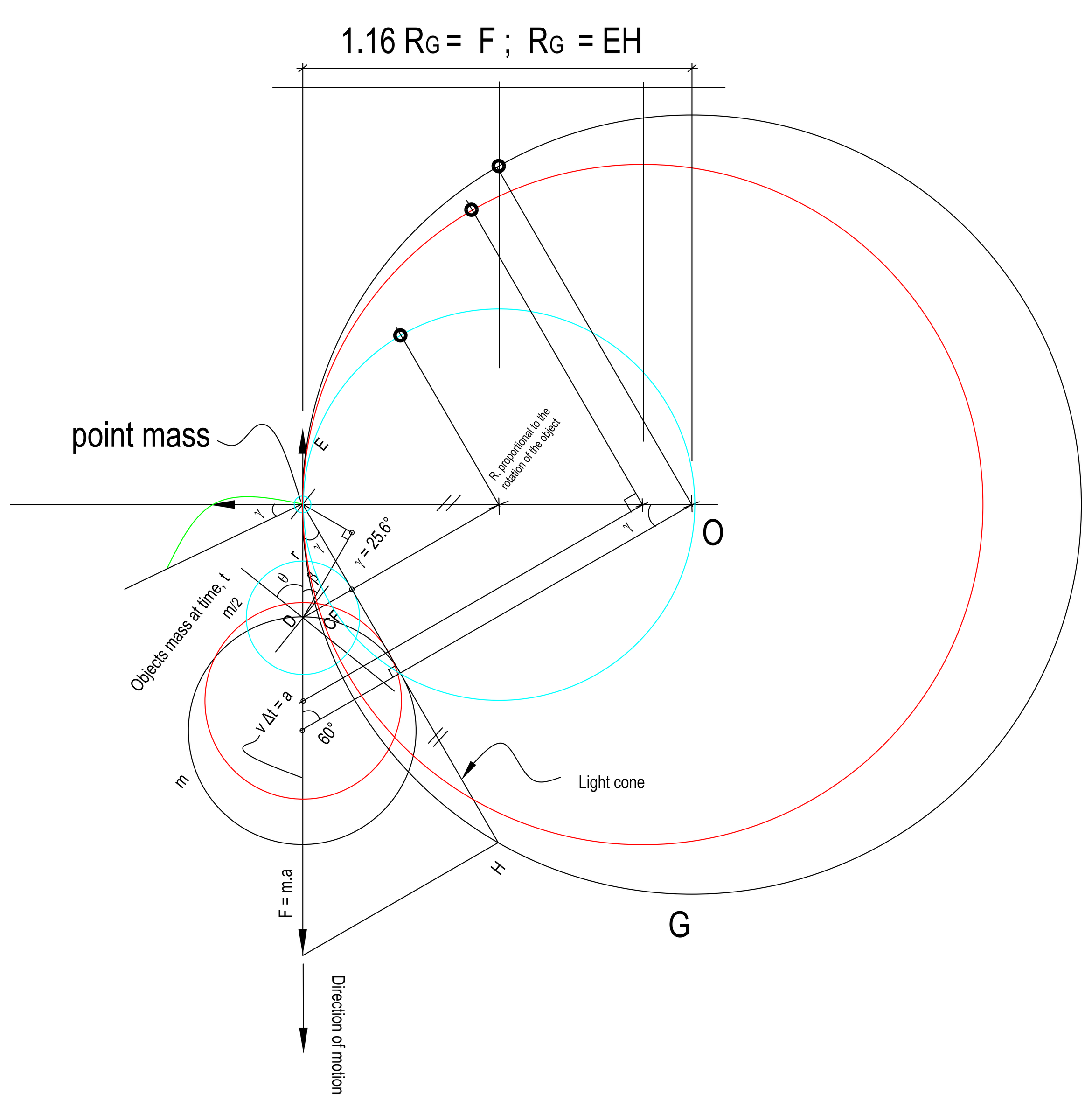
F = m.a vectorially in magnitude a = v
Also F = K x with x taken equal to the radius of the circle, m.
The total maximum energy E then for this mass
$E = 2\, (1/2\, k\, x^2) = 2\, (1/2\, m\, v^2)$
Eventually as the curvature becomes large $E = m c^2$
In essence, we keep time Δt constant and increase the velocity/curvature.

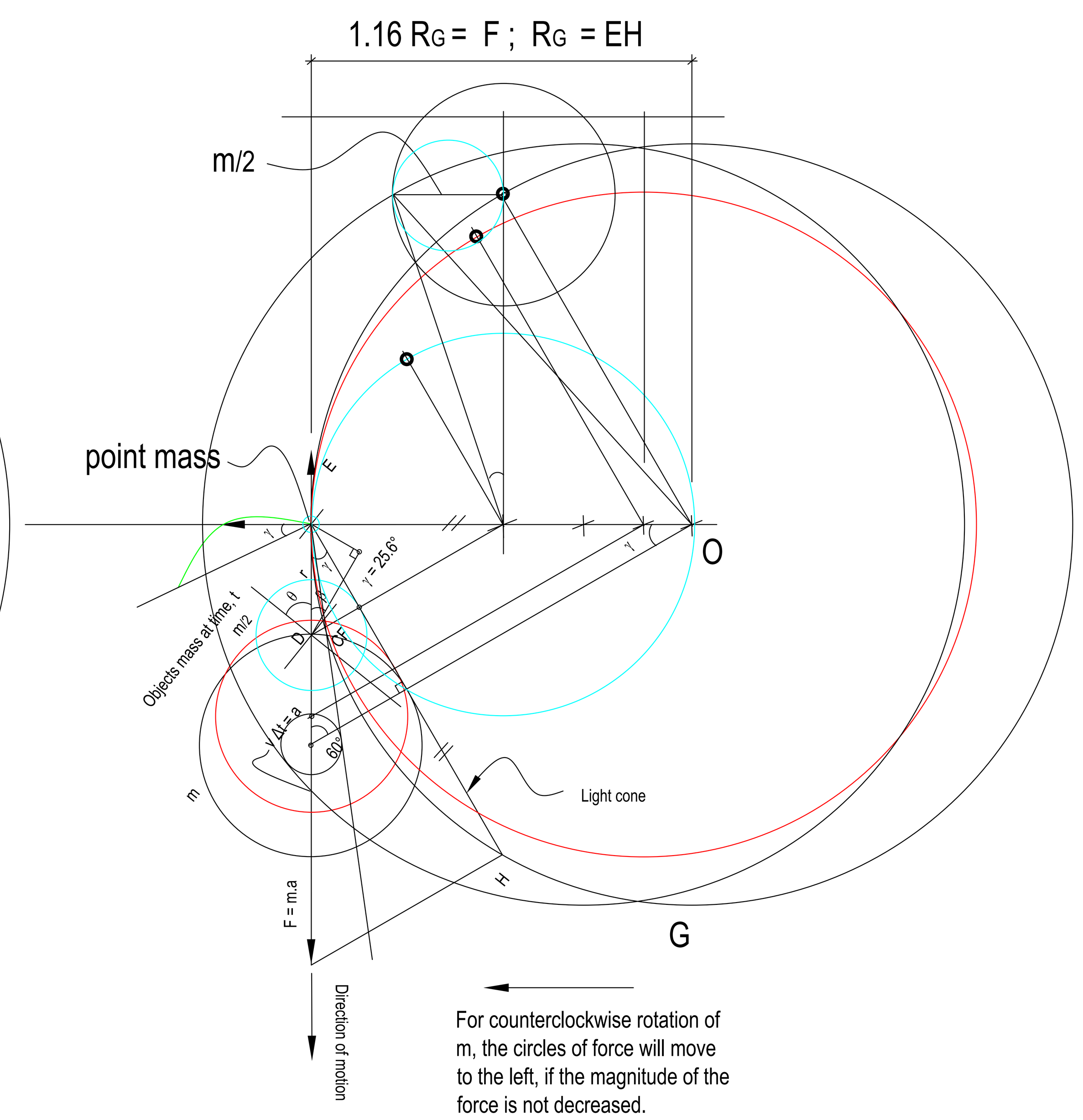
If the particles spin about its own axis increases, so does the curvature.

$E = m C^2$

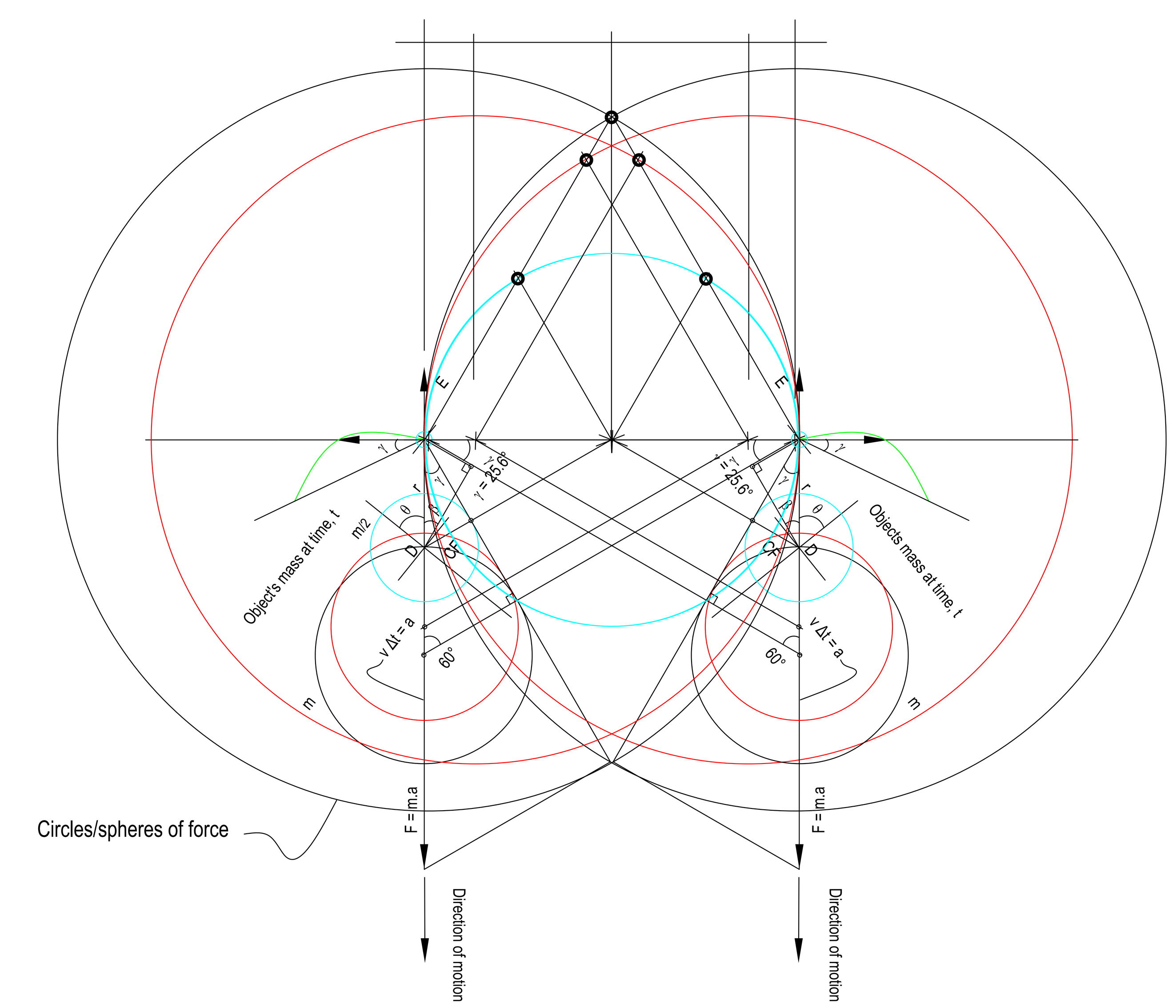
$1.16\, R_G = F\, ;\ R_G = EH$

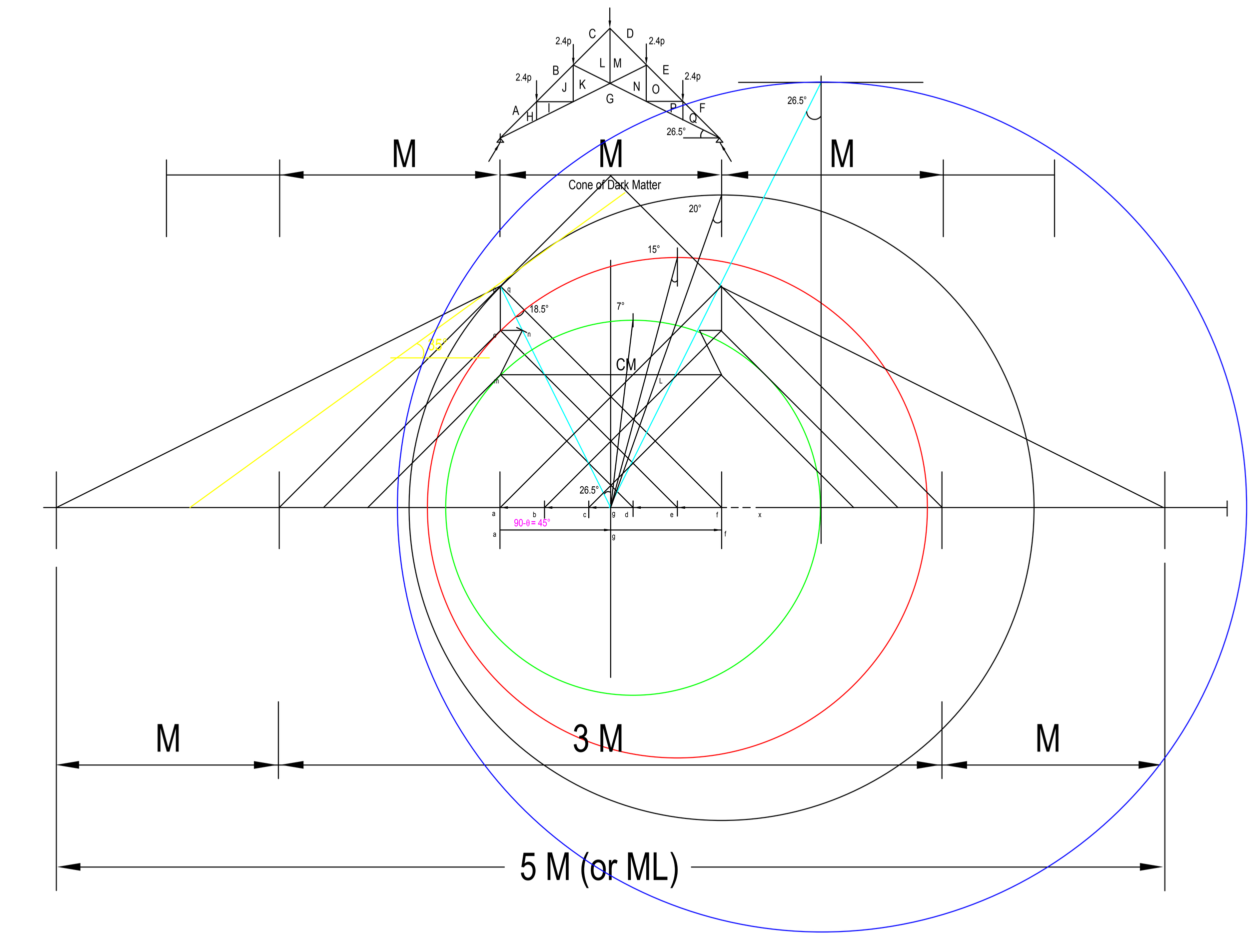
Move the circles to the left by m/2





F / M fluctuates in proportion to the spin of mass m about its center of gravity



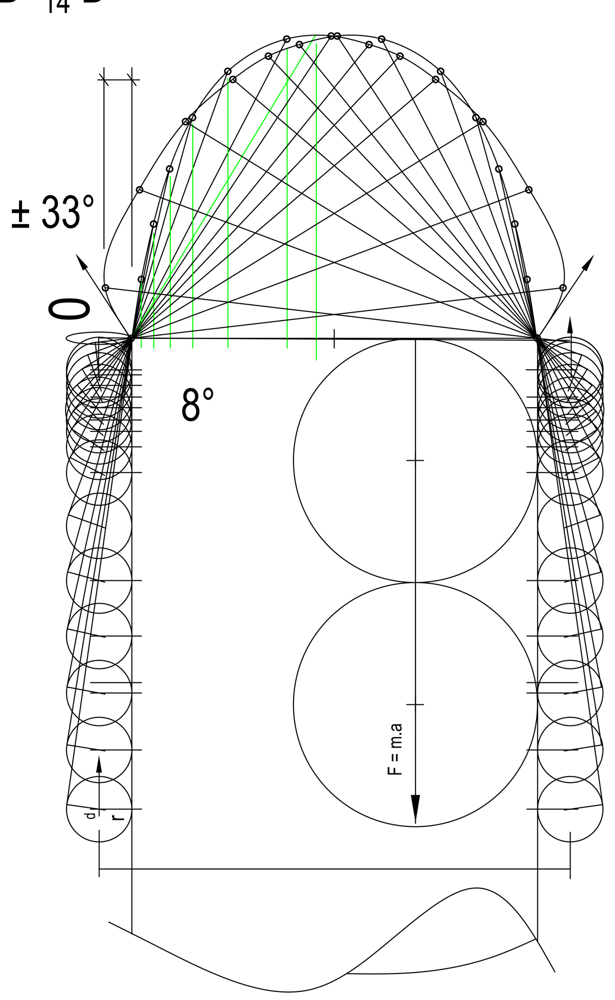


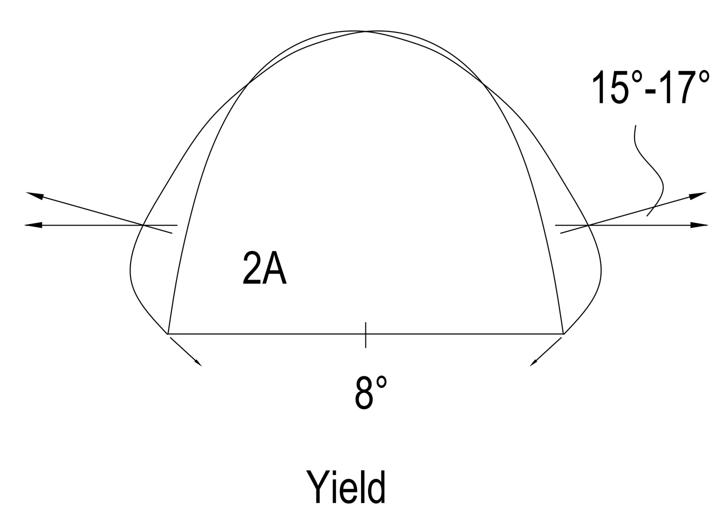
Spin of the mass about its center of gravity has a direct effect on the force F.

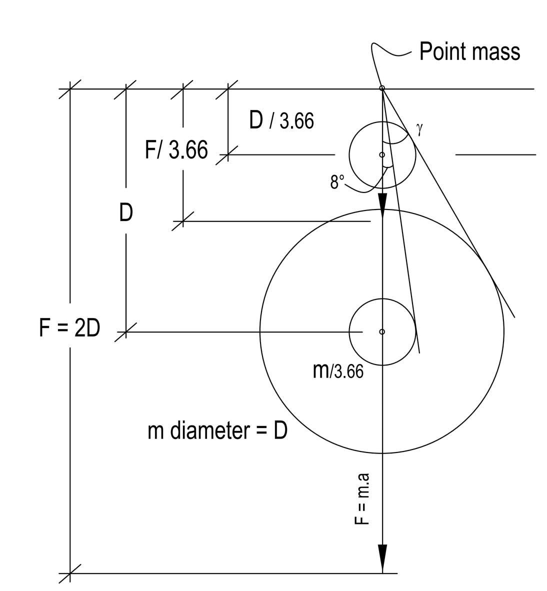
If the mass rotates about its center of gravity, then the circles/spheres of force would separate.
For the system to be irrotational, the circles whould separate as per the scissors truss diagram to the right. If the rotation is counter clockwise there will be a decrease in force. If there is no decrease in force F, then the circles would move to the left.

The reason why mass starts to rotate, columns buckle, etc...- and why we rotate the reference frame

0.068 D = 0.07D = $\frac{1}{14}$ D







If we take γ as 8°, the mass will be 3.66 times less. keep the mass at this level and draw the velocity pole curve.

Point at which mass starts to spin with the corresponding force F / 3.66 ≈ F / 4

As this is a plane problem, in two dimensions there will be two such forces for a total of F / 2.

Hence the process is irrotational the total force will equal F + F /2 = 3 F /2 = 3 D

Enlarge the segment by varrying θ and take it passed the yeild point, all the way up to failure, until the projectile has left the sphere (earth). Then draw the limit diagram as shown to the left.