

# Natural Frequency of Equilateral Polygons and Solids

Natural frequency of equilateral polygons inscribed in a circle:

Take a square inscribed in a circle, as the particle travels from E to E', its acceleration will equal to the radius of the circle.

This is the centrifugal force pulling on the radius. if the particle travels once around the circle, then this force will equal to four times the radius of the circle  $F = 4r$ .

If we travel more than  $90^\circ$  this force will be larger than  $r$  and the circle and square will translate in the direction of the tangential velocity  $V$  at that point.

Hence the natural period of vibration of the square inscribed in a circle is  $2\pi/4 = \pi/2$ , and its natural frequency is  $2/\pi$ .

In order for the square to remain stationary while we go once around its periphery, the string radius resisting the centrifugal force will have to be as strong as  $4r$ .

Imagine the center of the circle to lie on a pole and we revolve once around. Then on the joint where the wheel meets the pole there will be a force equal to  $4r$ .

For a sphere, a cubic, very rough, approximation will give  $6 \times 4r = 24r$  since the cube has six faces. The period of vibration will be  $6 \times 2\pi/4 = 3\pi = 9.42$ , and its natural frequency will equal  $1/3\pi$ .

Taking into account that the particle will have to travel along the diagonal  $ga$  as well, once it has made its revolution about the three faces connecting to one vertex, then with the edge of the cube equal to  $a$  we have the following proportion: ( $2r$  (additional length of travel on the diagonal) =  $a\sqrt{3}$  and)

To travel  $4a$  its period is  $\pi/2$ , what is the period if it traveled  $a\sqrt{3}$ ?

$4a / a\sqrt{3} : \pi/2 / x$

Solving for  $x$  gives  $x = (a\sqrt{3} / 4a) \cdot \pi/2 = \sqrt{3}/4 \cdot \pi/2 = 0.68$  in addition to the  $3\pi$ .

$T = (0.68) + 3\pi = 10.1$  and its natural frequency is  $0.099$  (The gravitational constant on the surface of a sphere metric system)

if we count 4 diagonals connecting the eight vertices of the cube, which is a more stable structure, then:

$T = 4(0.68) + 3\pi = 2.72 + 9.42 = 12.14$  and its natural frequency is  $0.08$

The period for steel was calculated for steel as 13!

The factor 10 is as the natural frequency is actually, the gravitational constant on the surface of a sphere.

Note, that in the calculation above we have only taken the vibrational degree of freedom of the particle which corresponds to its translation and not the rotational degree of freedom. If in addition we were to take into account the rotational degree of freedom of the particle, (about its centroid, related to its moment of inertia) then the cube's natural frequency would approach zero as the particle traveled to the opposite vertex  $a$ , at which point its direction of spin changes as it travels on the remaining 3 faces. The spins at  $g$  and  $a$  are equal and opposite.

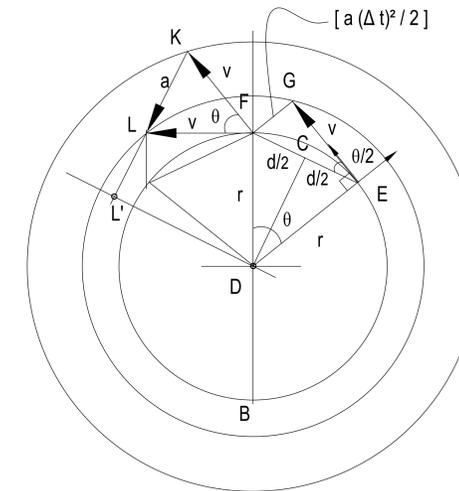
This constant would not change for calculations performed in non-Euclidean (skewed) coordinate systems, but would change for spaces of higher dimension (5 coordinate axes instead of three).

Hence, to make a material which is invisible to radar, we would have to have a coating which when struck by the radio waves would absorb the energy of the waves and reducing the coatings natural frequency to zero. The radio waves in this case, would not reflect back. This process would be momentary.

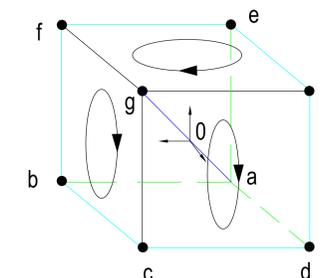
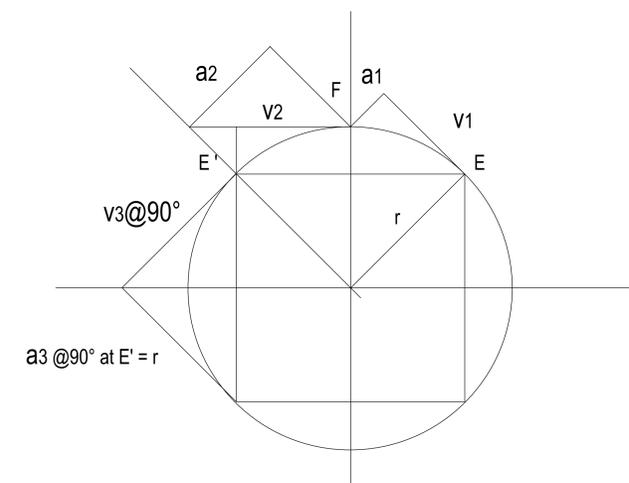
Repeat this process for shapes such like a triangle (tetrahedron) or pentagon (dodecahedron).

Eventually our goal will be to arrange molecules first in cross section then in three dimensions and find the natural frequency of the system.

For example changing the segment length by taking a large square and a small one their natural frequency will be proportional to the radii.



Recall when traveling from E to F in the diagram to the left, our acceleration  $a$  acts at C with the magnitude shown. if  $\theta$  equals 60 degrees, then  $a$  equals  $v$  which equals  $EF \cos \theta$ .



Having rotated about the 3 faces, the particle travels on the diagonal from  $g$  to  $a$ , reverses its spin and travels on the remaining 3 faces