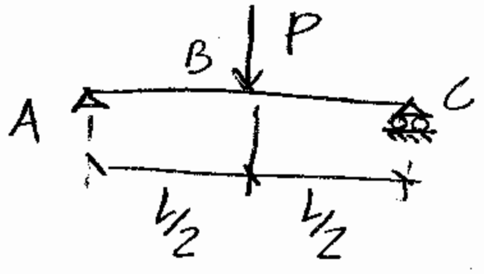
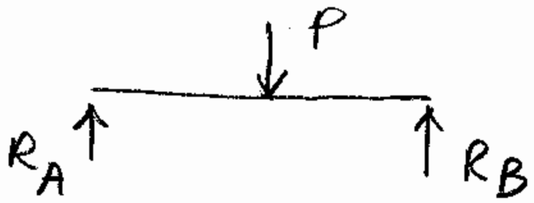
CLASS NOTES
LECTURE #2 & 3
(Pg. 1 → Pg. 11)
SIMPLY SUPPORTED BEAMS
BRACKETS, BRACINGS
(09/08/2005)

EXAMPLE: SIMPLY SUPPORTED BEAM LOADED W/ CONC. LOAD P, FIND REACTIONS.



DRAW FREE BODY DIAGRAM F.B.D.



$\sum F_x = 0$

$\sum F_y = 0$

$\sum M = 0 \rightarrow$ TAKE MOMENT $\sum$ ABOUT ONE OF SUPPORTS

$\Longrightarrow$ ELIMINATE ONE UNKNOWN REACTION.

BY OBSERVATION, SINCE SYM. LOADING $\rightarrow$ REACTIONS WOULD HAVE TO BE EQUAL. $R_A = R_B$ & $= P/2$

AS $\sum F_y = 0$.

BRUTE FORCE METHOD:

$\sum F_x = 0$ AS ONE END SITS ON ROLLER $\Longrightarrow C_x = 0$

AS $C_x = 0 \Longrightarrow A_x = 0$ SINCE $\cancel{C_x} + A_x = 0$

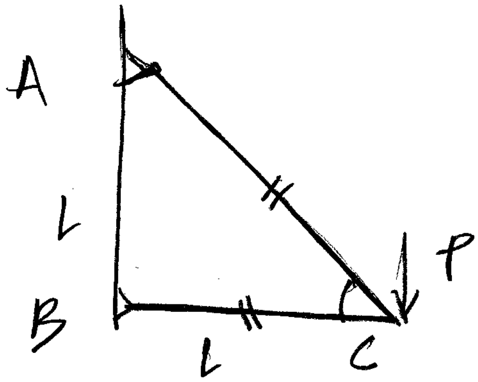
$\sum F_y = 0 \Rightarrow R_A + R_C - P = 0 \longrightarrow R_A + R_C = P$ EQ(1).

$\circlearrowleft+$ $\sum M_A = 0$ RIGHT HAND RULE (SCREW)

$$\underbrace{-P}_{\text{FORCE}}\underbrace{L/2}_{\text{DISTANCE}} + R_C L + R_A(0) = 0 \Longrightarrow R_C \cancel{L} = P\cancel{L}/2 \therefore \boxed{R_C = P/2}$$

PLUG IN EQ(1): $R_A = P - R_C = P - P/2 = \boxed{P/2}$

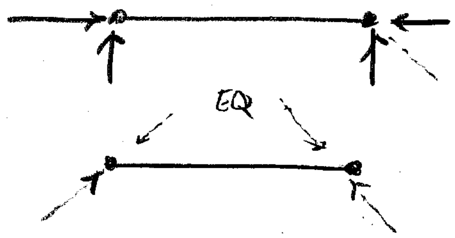
SOLVE A BRACKET:



TWO FORCE MEMBER

3 FORCE MEMBER.

2 FORCE MEMBER ie;

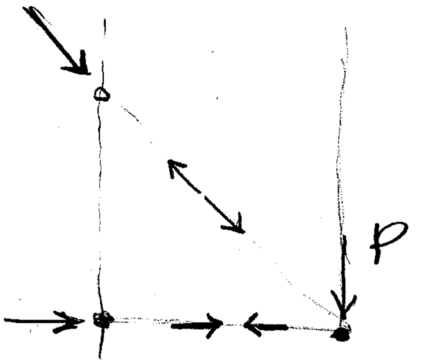


TRUSS MEMBER W/ PINS.

COMPRESSION OR TENSION.

BOTH MEMBERS AC & BC ARE TWO FORCE MEMBERS

EXTERNAL REACTIONS @ A & B WOULD HAVE TO MEET P @ A POINT.

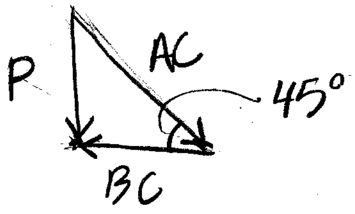


& THAT POINT IS C

$\Rightarrow$ BC IS IN COMPRESSION.

AC IN TENSION.

GRAPHIC POLYGON.



$\sin 45° = P/AC$

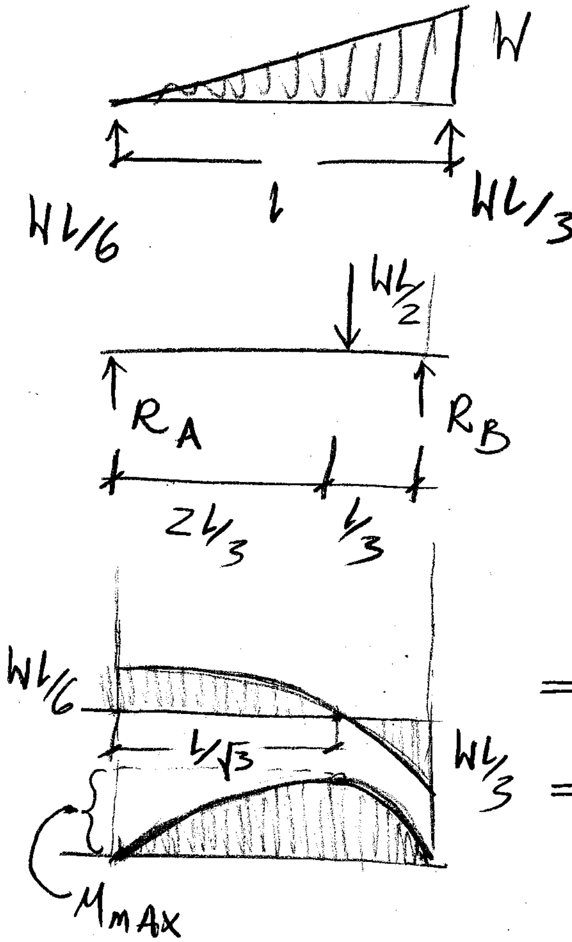
$AC = \dfrac{P}{\sin 45°} = \dfrac{P}{\frac{\sqrt{2}}{2}}$

$\sin 45° = \dfrac{\sqrt{2}}{2}$

$AC = \dfrac{2P}{\sqrt{2}} = \dfrac{\cancel{2}P\sqrt{2}}{\cancel{2}}$

$= P\sqrt{2}$

$\boxed{BC = P}$



$\Sigma F_x = 0$ ✓

$\Sigma F_y = 0 \Longrightarrow R_A + R_B = WL/2$

$\circlearrowleft + \Sigma M_A = 0 \Longrightarrow -\frac{WL}{2} \cdot \frac{2L}{3} + R_B L = 0$

$R_B = WL/3$

$\Longrightarrow R_A = WL/2 - WL/3 = WL/6$

$V(x) = R_A - \frac{Wx}{L} \cdot \frac{x}{2}$ WHEN $V(x) = 0$

$\Longrightarrow V(x) = \frac{WL}{6} - \frac{Wx^2}{2L} = 0$

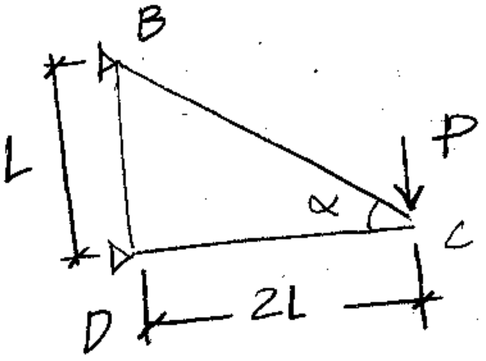
$\Longrightarrow \frac{x^2}{2L} = \frac{L}{6} \Longrightarrow x^2 = \frac{L^2}{6} \Longrightarrow x = \frac{L}{\sqrt{3}}$

$M(x) = R_A x - \frac{WL}{6}x \cdot \frac{x}{2} \cdot \frac{1}{3}x = \frac{WL}{6}x - \frac{Wx^3}{6L}$

$M_{MAX} = M\left(\frac{L}{\sqrt{3}}\right) = \frac{WL}{6} \cdot \frac{L}{\sqrt{3}} - \frac{W}{L} \frac{L}{\sqrt{3}} \cdot \frac{L}{2\sqrt{3}} \cdot \frac{1}{3} \frac{L}{\sqrt{3}}$

$= \frac{WL^2}{6\sqrt{3}} - \frac{W}{\sqrt{3}} \cdot \frac{L^2}{18}$

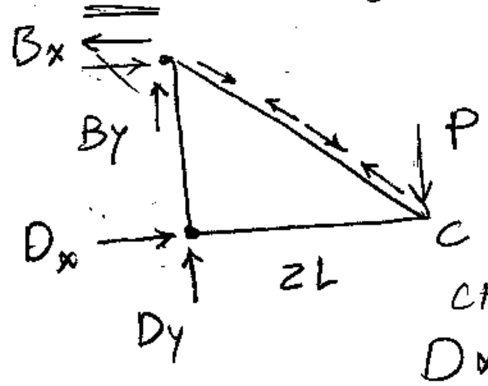
$= \frac{WL^2}{6\sqrt{3}} - \frac{WL^2}{18\sqrt{3}} = \frac{3WL^2 - WL^2}{18\sqrt{3}} = \frac{WL^2}{9\sqrt{3}}$



FBD: $\sum M_D = 0 \quad -P \times 2L - B_x L = 0$

$B_x = -2P$ (4)

$\sum F_x = 0$



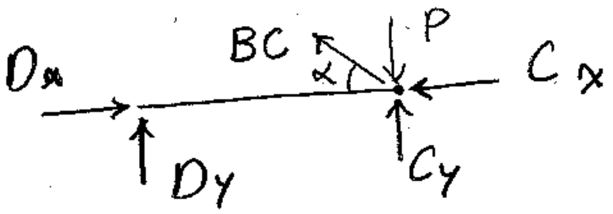
$\Rightarrow B_x - D_x = 0$

$\Rightarrow B_x = D_x$

CHANGE ARROW B_x ($\leftarrow$)

$\boxed{D_x = 2P}$

TAKE ELEMENT DC FBD



$\alpha = \tan^{-1}\left(\frac{L}{2L}\right) = \tan^{-1} 0.5 = 26.56°$

AT JOINT C: $\sum F_y = 0$

$-P + BC \sin\alpha + C_y = 0$ (1)

$\sum F_x = 0 \Rightarrow \quad -BC\cos\alpha + D_x = 0$

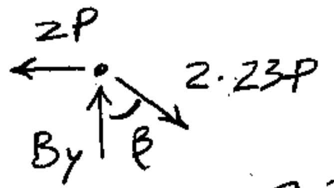
$\Rightarrow \quad -BC\cos 26.56 + D_x = 0$ BUT $D_x = 2P$ ($\rightarrow$)

$D_x = BC(0.89) \Rightarrow 2P = BC(0.89) \Rightarrow BC = 2.23P$

(1) $-P + BC \sin 26.56 + C_y = 0$

$\Rightarrow -P + 2.23(0.4471)P + C_y = 0 \Rightarrow \boxed{C_y = P}$

JOINT B:



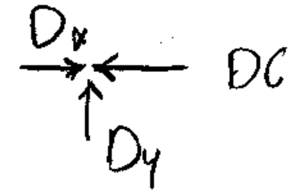
$\sum F_y = 0 \Rightarrow -2.23P\cos\beta + B_y = 0$

$\Rightarrow B_y = 2.23P\cos\beta$ BUT $\beta = 180 - 90 - \alpha$

$\beta = 90 - \alpha = 90 - 26.56 = 63.44°$

$\Rightarrow B_y = 2.23P(\cos 63.44) = P$

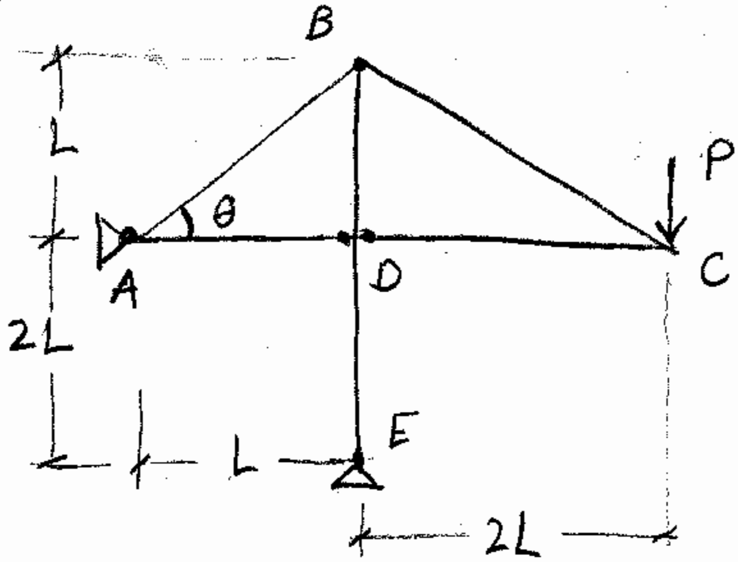
JOINT D:



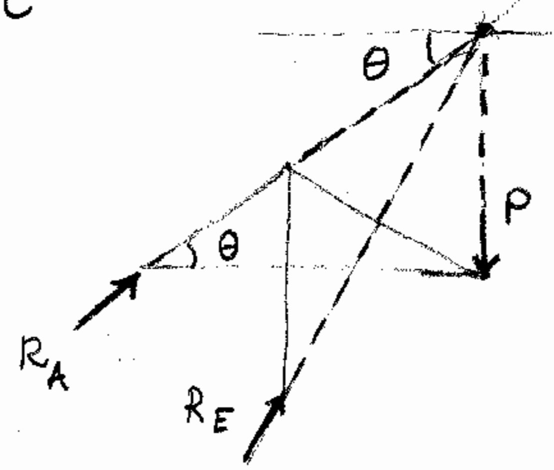
$\sum F_x = 0$

$\sum F_y = 0 = D_y$ QED

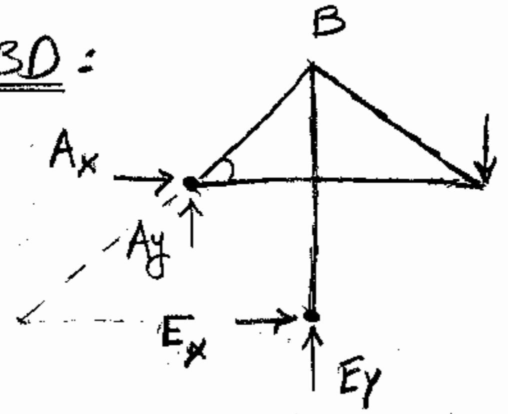
PROBLEM:



GRAPHIC METHOD:

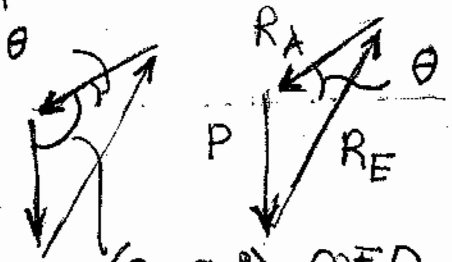


FBD:



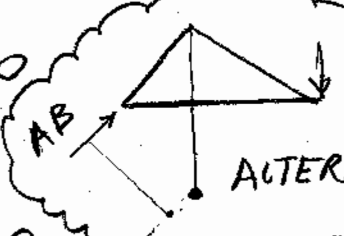
$\theta = \tan^{-1}(L/L)$
$= 45°$

R_E COMPRESSION
A_A TENSION $\Rightarrow$ REVERSE



$(\theta + 90°)$ QED

$\Sigma F_y = 0 \Rightarrow A_y + E_y - P = 0$
$\Rightarrow A_y + E_y = P \quad (1)$
$\Sigma F_x = 0 \Rightarrow A_x + E_x = 0$
$\Sigma M_E = 0 \qquad A_x = -E_x \; (2)$
$-P\,2L + A_y \times L + A_x\,2L = 0$
(3)



$\Sigma M_A = 0$
$-P\,3L + E_y L + E_x 2L = 0 \quad (4)$

PLUG (2) INTO (3)
$-2PL + A_y L - E_x = 0 \quad (6)$

PLUG (1) INTO (4) $\Rightarrow E_y = P - A_y$ INTO (4) $\Rightarrow -3PL + PL - A_y L + 2E_x L = 0$
$- A_y L + 2E_x L = 2PL \quad (5)$
$\Rightarrow A_y - 2E_x = 2P \quad (6)$

$$-PL(2) + A_y l + A_x 2l = 0 \quad (EQ\ \#3)$$
$$-3Pl + E_y L + E_x 2l = 0 \quad (EQ.\ \#4)$$

$$\left.\begin{array}{l} (3)\ A_y l + A_x 2l = 2PL \\ (4)\ E_y L + \cancel{E_x 2l} = 3PL \\ (6)\ A_y L - \cancel{2E_x l} = 2PL \end{array}\right\} \Rightarrow E_y l + A_y L = 5Pl$$

$$\Rightarrow E_y + A_y = 5P$$
$$E_y + A_y = P$$

$$\left.\begin{cases} A_y + E_y = P \\ \cancel{A_y} + 2A_x = P \end{cases}\right\} \Rightarrow E_y - 2A_x = 0 \Rightarrow E_y = 2A_x$$

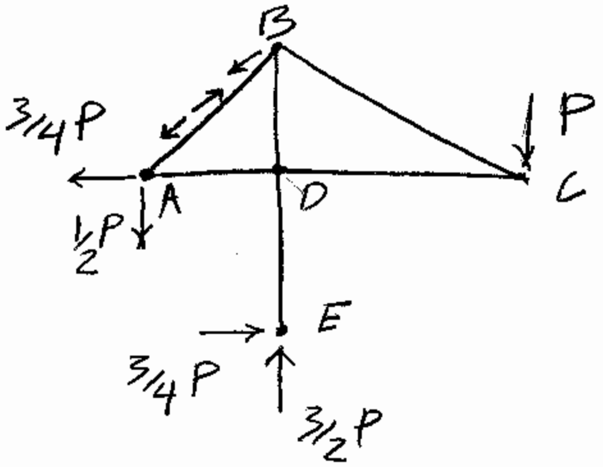
$$* E_y + 2E_x = 3P \quad * A_x = -E_x$$

$$\begin{cases} E_y + 2E_x = 3P \\ A_y + 2A_x = P \end{cases}$$

$$E_y - 2A_x = 3P \qquad A_x = -E_x$$
$$E_y = 2A_x \qquad A_x = E_y/2$$
$$\Rightarrow E_y/2 = E_x$$
$$\Rightarrow \boxed{E_y = 2E_x}$$

$$E_y + 2E_x = 3P \Rightarrow 2E_x + 2E_x = 3P \rightarrow 4E_x = 3P$$
$$\Rightarrow \boxed{E_x = 3/4\,P} \quad \boxed{A_x = -3/4\,P}$$

$$E_y = 2E_x = 2(3/4\,P) = \boxed{3/2\,P} \quad A_y = P - 3/2\,P = \boxed{-1/2\,P}$$

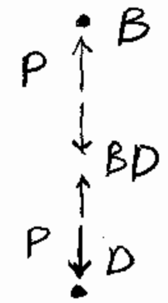


FIND FORCE IN MEMBER DE. (7)

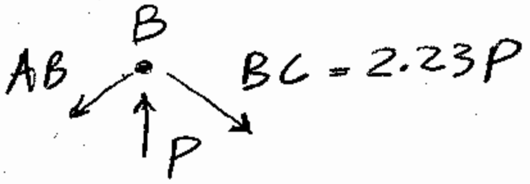
FROM PREVIOUS PROBLEM.

$B_y = P\uparrow \quad D_y = 0$

$\Rightarrow$



TAKE JOINT B:



$AB = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(-\tfrac{3}{4}P\right)^2 + \left(\tfrac{1}{2}P\right)^2} = \sqrt{0.5625 + 0.25} = 0.9P$

$A_y = -\tfrac{1}{2}P \; , \; BC_y = -2.23P \times \cos 63.44 = -P$

D

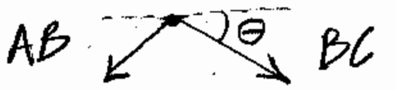
$\tfrac{1}{2}P\downarrow \; \uparrow_P \; \downarrow P = \tfrac{1}{2}P\downarrow$ FROM FOUNDATION $\tfrac{3}{2}P\uparrow$

$= P\uparrow$ WHICH BALLANCES WITH $P\downarrow$ EXT. FORCE.

@ JOINT D. THEREFORE RIGHT SIDE SHEAR = $P\downarrow$

LEFT SIDE SHEAR = $-\tfrac{1}{2}P$ OR $\tfrac{P}{2}\downarrow$

FOR X DIRECTION TAKE JOINT B.



$2.23P \cos 26.56 - 0.9P \cos 45$ SHOULD = 0

$2P - 3/4 P = 1.25P \; \rightarrow$

$\rightarrow 1.35P$

$AD = 3/4 P \quad 2P = D_x$

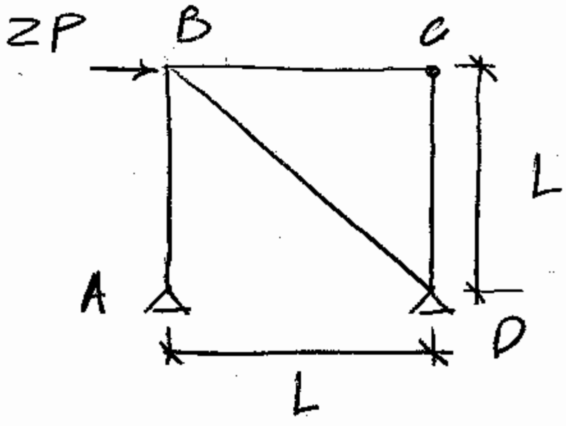
$\rightarrow \cdot \leftarrow$

JOINT D: X DIR $\quad D_x = 2P$

$2P - 0.75P = 1.25P \; \leftarrow$

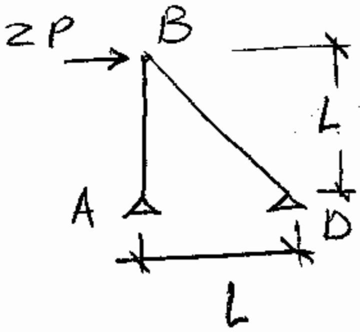
$= 0$

$\Rightarrow$ LOOKING AT COLUMN EB THERE IS A MOMENT AT JOINT D $\quad M = 1.25P \times L$



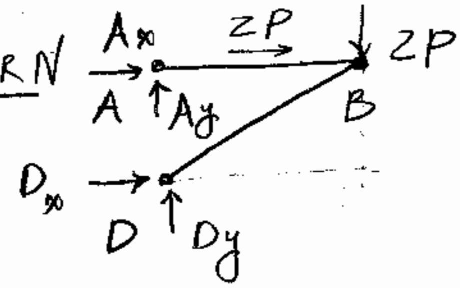
BRACING:

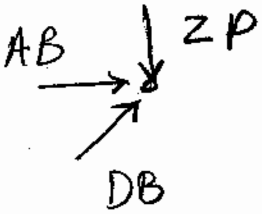
NOTE AS MEMBERS BC & CD @ JOINT C HAVE NO FORCE $\Longrightarrow$ THEY ARE ZERO FORCE MEMBERS.



- GET RID OF THEM & WE HAVE A BRACKET.

TURN





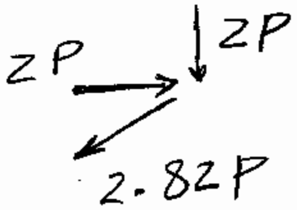
DB IS A TWO FORCE MEMBER $\Longrightarrow$ @ JOINT B

$DB \sin\theta = -2P \Longrightarrow DB = -2P/\frac{\sqrt{2}}{2} = \frac{-2P}{0.7071}$

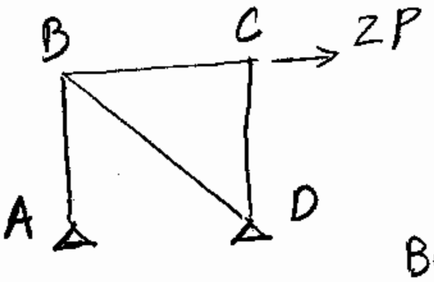
$DB \cos\theta + AB = 0 \qquad DB = -2.8284$

$DB\,(0.7071) = -AB \Longrightarrow \cancel{-}2.8284\,(0.7071) = \cancel{-}AB$

$AB = 2P$

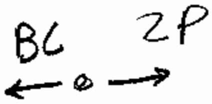


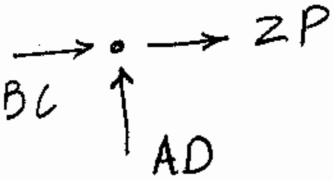
REVERSE ARROW FOR DB.



IF LOAD IS MOVED TO JOINT C:

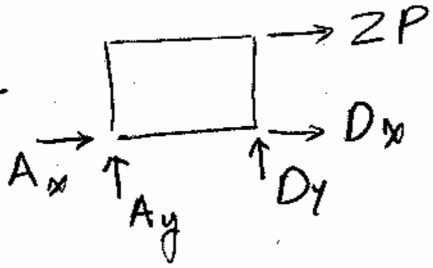
AT JOINT C: $\Sigma F_x = 0$





$BC + 2P = 0 \Longrightarrow BC = -2P$

GLOBAL STRUCTURE



$\Sigma F_x = 0$

$\Sigma F_y = 0$

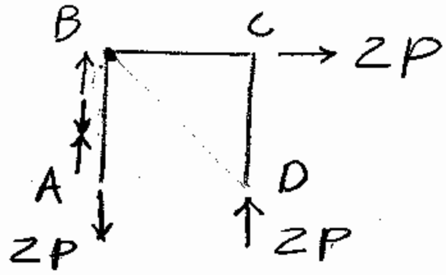
$\Sigma M_A = 0$

$\sum F_x = 0 \Rightarrow A_x + D_x + 2P = 0$

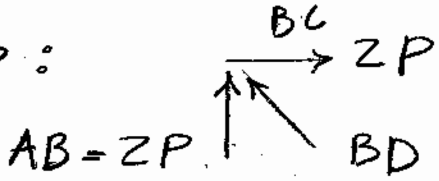
$\sum F_y = 0 \Rightarrow A_y + D_y = 0 \Rightarrow A_y = -D_y$

$\sum M_A = 0 \Rightarrow -2P\cancel{L} + D_y\cancel{L} = 0 \Rightarrow D_y = \boxed{2P} \uparrow$

$\rightarrow A_y = -2P \downarrow$



AT JOINT B :

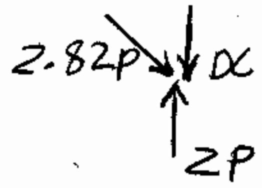


WE KNOW $BC = 2P$
& $AB = 2P$

$\Rightarrow$ BD WOUD HAVE TO PUT JOINT B IN EQUILIBRIUM.

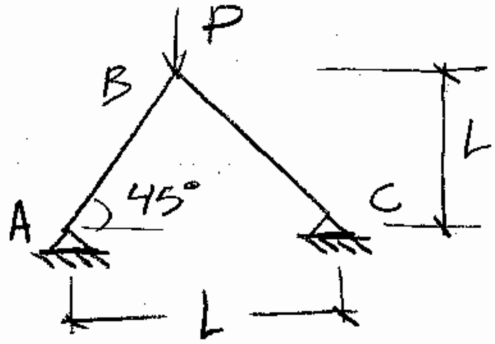
$BD = \sqrt{(2P)^2 + (2P)^2} = \sqrt{8P^2} = 2.8284P$

& AT JOINT D

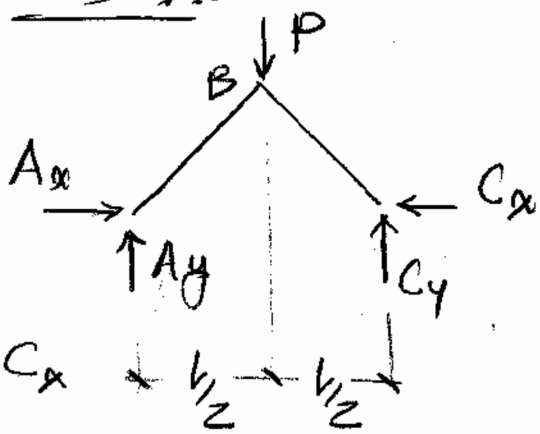


WE SEE : MEMBER DC TAKES NO LOAD. $\underline{DC = 0}$

ARCHES:



F.B.D.:



$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow A_x - C_x = 0 \Rightarrow A_x = C_x$

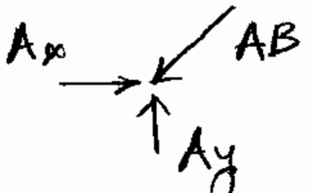
$+\uparrow \sum F_y = 0 \Rightarrow A_y + C_y - P = 0$

$\circlearrowleft + \sum M_A = 0 \Rightarrow -P\,L/2 + C_y L + \cancel{A_x(0)} + \cancel{A_y(0)} + \cancel{C_x(0)} = 0$

$\Rightarrow \boxed{C_y = P/2}$

$\sum F_y = 0 \Rightarrow A_y = P - C_y = P - P/2 = \boxed{P/2}$

MEMBER AB IS A TWO FORCE MEMBER $\Rightarrow$ AT JOINT A:



$\uparrow A \ \sum F_x = 0$

$-AB\cos\theta + A_x = 0 \Rightarrow A_x = 0.7071\,AB$

$\sum F_y = 0 \Rightarrow -AB\sin\theta + A_y = 0$

$\Rightarrow A_y = 0.7071\,AB$ BUT $A_y = P/2$

$\Rightarrow AB = A_y / 0.7071 = \frac{P}{2}\left(\frac{1}{0.7071}\right) = \frac{P}{\cancel{2}}\,\frac{\cancel{2}}{\sqrt{2}}$

$= \frac{P\sqrt{2}}{2}$

$\sin 45 = \frac{\sqrt{2}}{2}$

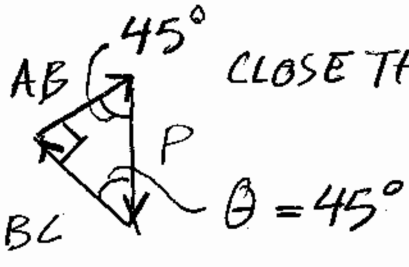
$\cos 45 = \frac{\sqrt{2}}{2}$

$\sqrt{2}/2 = 0.7071$

$AB_x = P\sqrt{2}/2 \times \sqrt{2}/2 = \frac{P(2)}{4} = \boxed{P/2}$

$AB_x = A_x$; $C_x = P/2$ AS WELL . QED

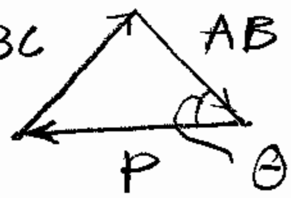
GRAPHIC METHOD:

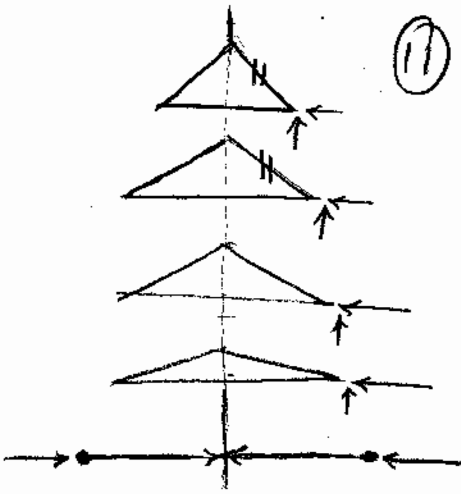


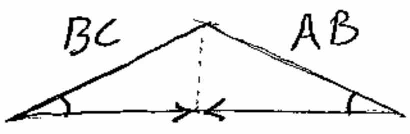
CLOSE THE POLYGON.

$\frac{\sin 45°}{AB} = \frac{\sin 45°}{BC} = \frac{\sin 90°}{P}$

ROTATE GRAPHIC
& THINK OF P
AS THE SPAN
OF ARCH.





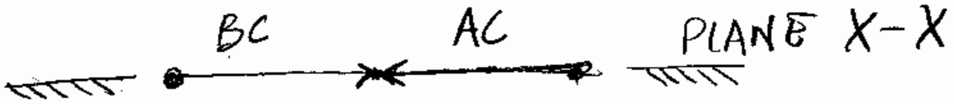


AS WE DECREAS ANGLE θ
SPAN INCREASES

$P \propto L$

LOAD P IS PROPORTIONAL TO SPAN L. NEXT THINK OF BC & AB AS PROTECTION OF LOAD OR THRUST AT SUPPORT OF ARCH.

AS WE DECREASE ANGLE MORE & MORE WE REACH A FLAT LINE



THIS SHOULD BE THE MAXIMUM FORCE COMPRESSIVE WHICH THE MEMBERS HOULD ME DESIGNED FOR & AT THIS POINT THE ARCH BECOMES A BEAM. & THE SHEAR FORCE BECOMES A FACTOR IN DESIGN. + BENDING MOMENT.

AS WE GO BELLOW THE PLANE X-X, THE ARCH BECOMES A CABLE & THE REVERSE IS TRUE & VERTICAL & HORIZONTAL FORCES AT THE SUPPORT FORM INTO COMPRESSIVE AXIAL LOAD. $\frac{P}{2}$ @45°