Introduction to Strength of materials: (Chapter-2) Class #3, #4 – September 20, 27

Introduction to Stress and Strain, and mechanics of deformable bodies
Stress: Load/Area or Force/Area [F/L.L] [kips/square inch]
Where F=Force (pound force) and L=Length (inches or feet for SI units)

Strain: Change in length over original length of the member under tensile or compressive load. As we load the material, the material will extend if the extension we all delta L, ΔL, or change in length, then the change in length divided by the original length will give us the strain in the material

$\varepsilon = \Delta L/L$ [Unit of Length/Unit of Length] and therefore it is a dimensionless quantity.

To establish a relationship between stress and strain, we can perform tests on a given specimen, one such test is the tensile test, where the specimen is loaded in tension in a machine (Tensile Test Machine) and the stress and strain are recorded. The graph looks like the one in figure bellow: (Note: Tests including but not limited to tensile tests on crystals of materials have also been performed using other mechanical means which we will cover later)

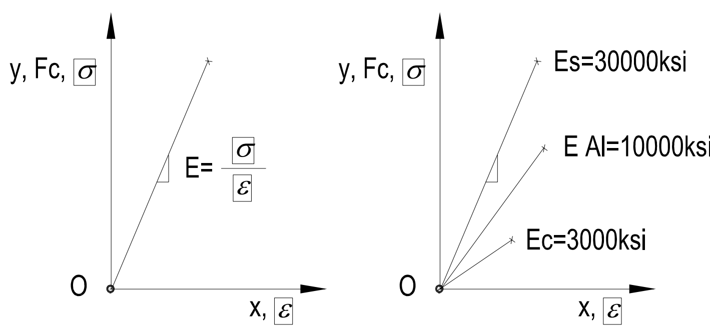

Figure-1: Relation between Stress and Strain

The slope of the stress strain Curve is termed the Modulus of Elasticity. Or Young's Modulus. And the relationship is called Hooke's Law as he was the first person to observe that there is a linear relationship between stress and elongation of a bar in tension.

From the graph it can be observed that the stronger the material the greater the slope, so in Figure-1 wee see that steel is the strongest material Es, then Aluminum, (E_{AL}), and then Ec Concrete.

Looking at a typical stress strain curve, when the specimen is loaded, it will deform and elongate as shown below in a linear fashion with a slope E up to the proportional limit. At this point the material if loaded further will start to yield. If unloaded the specimen will return to its origin O. (Approximately, with a 0.2% offset)

After the yield the material is in plastic region and if unloaded the material will not return to its original length and will have what is called a permanent deformation or permanent set. Passed the elastic range in Figure-3, each time we reload then the material becomes stronger but its elastic limit/point decreases if loaded to the elastic limit and becomes less ductile.

During the plastic deformation and continued loading the grains of the material rearrange themselves to try to resist the load and stress in what is called strain hardening.

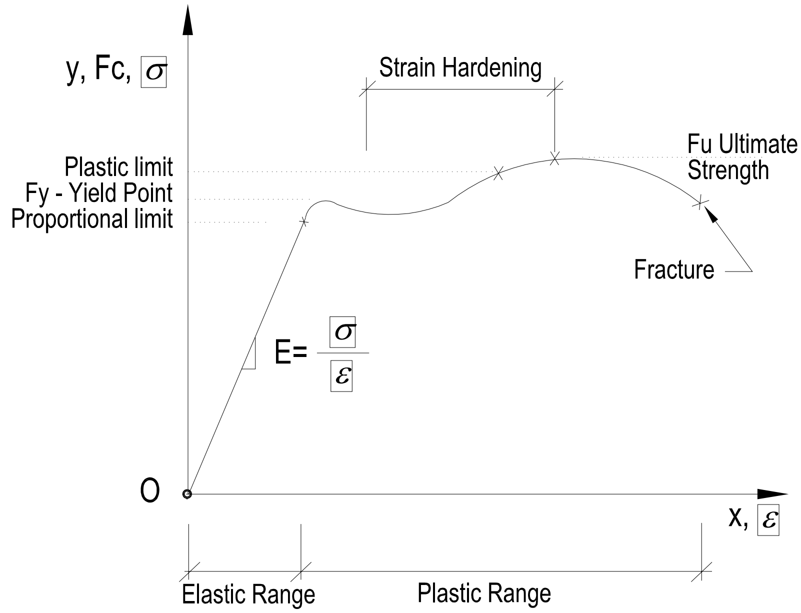

Figure-2: Stress Strain Curve

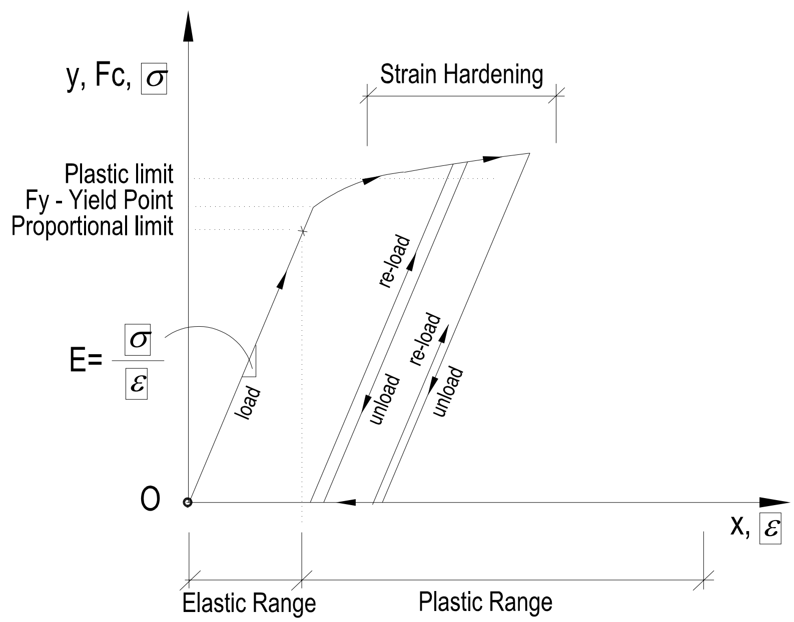

Figure-3: Strengthening during loading and unloading in plastic range

Formulas Related to Stress Strain Curve:

$\sigma = F/A$ Stress = Force / Area

$\varepsilon = \Delta L/L$ Strain = change in length / Original length

$\sigma = E \cdot \varepsilon$ Stress = Elastic Modulus . Strain

See the following link PDF Drawing for further discussion of , Shear, Moment, and Stress:

www.gregorianengineers.com/BAC/fall2005/stressdwg.pdf

Terminology:

Factor of safety: Stress at Yield Plastic or Ultimate/ Factor, enabling the quantity to be used in design. (Factor dependant on uncertainty in determination of loading, testing of material) Allowable Stress a quantity and value determined to be used in design of members given the allowable loads.

Factor of safety for 36 Ksi steel in compression is taken as 2/3 yield stress = 2/3 x 36000 = 24ksi

Load Factors: Factors to increase loads stated in building codes based on uncertainty in method of quantifying loads. (For example, 1.4 Dead, 1.7 Live, 0.7 EQ., 1.0 Wind)

Ultimate Stress (determined while testing) The highest point in stress strain curve before fracture.

Ultimate Strength (determined by applying a factor of safety depends on the loading condition and type of member being loaded and importance of the member in structural stability, axial load on column, bending moment or shear in beam)

Material testing and mechanical properties, Hooke's Law, and Modulus of Elasticity. Tension test, where a specimen is loaded in tension and its elongation and strain.

Elastic Range: Range on the stress strain curve up to the elastic or proportional limit.

Elastic Limit=Proportional Limit: point on the stress strain curve up to which if load is removed the material will return back to its original length and shape

Yield Point, the point on the stress strain curve after which the specimen/material will not return to its original length and shape. The crystal structure of the specimen/material begins to change up to plastic limit.

Plastic-Strain hardening Range: Range between Yield and Ultimate Stress.

Plastic limit: In the plastic range, The specimen will be able to take additional load. 1.2 times the yield strength.

Ultimate stress: Maximum stress the specimen can be subjected to under testing. The highest point on the stress strain curve. The point on the stress strain diagram after which the material elongates without further additional load and elongates until fracture.

Toughness: Measure of how much force is required to deform the material to breaking point. The area under the stress strain diagram. Analogy: work = force x distance from physics. Charpy impact test (impact with drop hammer) used to determine toughness for ductile materials.

Hardness: Resistance of the material to penetration.

Resilience: The ability of the material to absorb energy

Fatigue: Materials response and failure in form of fracture under constant repeated loading.
S-N Curve for fatigue- Stress vs. Number of Cycles of applied load. (Example-Airplane wing and how long can it sustain dynamic stresses induced by vibration of the wing during flight before failure)

Examples:

Assuming a Force of 500 lbs acts on an area of 10 in^2 the stress will be equal to 500/10=50 in^2.

Increasing the area to 20 inches will decrease the stress to 500/20 = 25 in^2. If area is doubled the stress will be halved.

Let us suppose a wood column of 10"x10" in cross section is loaded with a 50,000 lb load P. The area of the column A=100 in^2.

σ = P/A = 50000/100 = 500 psi

If a 2x6 wood stud was loaded with the same force of 50000 lbs the stress would be:

A = 1.5"x5.5" (as the wood is cut to the nominal 1.5x5.5 dimension but specified as 2x6)
A= 8.25 in^2
σ = P/A = 50000/8.25 = 6060.606 psi

Problem: Compressive stress and strain of bracing member (buckling neglected)

Given steel double angles welded, back to back, 2L3x3x1/4, 20 feet long loaded in compression with 40,000 lbs, determine the stress in the members. Calculate the elongation in the member. Neglect buckling for the problem.

Use A36 Steel: Fy = 36000 psi = 36 ksi
E (Elastic Modulus) = 29000000 psi = 29000 ksi (Given for steel)
From AISC tables we can find the Area of the 2 angles as: A = 2.88 in^2
P = 40,000 lbs = 40 kips

σ = P/A	σ = 40/2.88 = 13.88 ksi
σ allowable = 2/3 x Fy	σ a = 0.67 x 36= 24 ksi
	13.88< 24 OK Member is not overstressed
	(Buckling failure should be checked)

Calculate Strain:
$\varepsilon = \Delta$ L/L $\quad \Delta$ L = ε . L

Formulas to use for problem of this kind (watch units):
σ = E . ε $\quad \varepsilon = \sigma$ / E $\quad \varepsilon$ =P/A.E $\quad \Delta$ L/L=P/AE $\quad \Delta$ L=PL/AE

For Strain: ε =P/A.E $\quad \varepsilon$ = (40/2.88) x (1/E) = 40/(2.88 x 29000) = 0.00048 in/in

Δ L = ε . L $\quad \Delta$ L = 0.00048 x 20 ft x 12"/ft = 0.1149 in ; Length reduced by 0.1149 inches

Analogy, Foundation Design and Geotechnical Engineering:

Preliminary sizing of rectangular concrete footing:
Assuming Bearing capacity BC=2 Tons/sq.ft. and given the following floor plan of 20’-0”x20’-0” grid, assuming an office building with total Live and Dead load of 150 lbs/sq.ft., the load on the column on centerline B-2 will be:
The tributary area times the floor load. The tributary area for the column is the area of the rectangle 20’-0”x20’-0”= 400 square feet.
P = 20’-0”x20’-0”x150 lbs./sq.ft.=60000 lbs.
BC = 2 Tons/sq.ft. = 4000 lbs/sq.ft. = 4 kips/sq.ft.
Area of footing = 60000/4000 = 15 square feet
If a square footing is chosen, then the footing dimension will be $\sqrt{15}$ = 3.873 feet or 4’-0”x4’-0”.

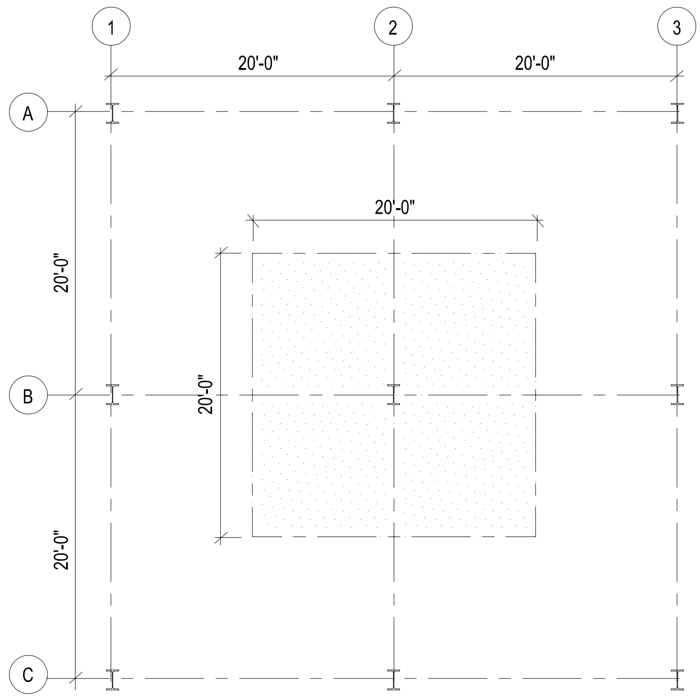

Figure-4

Geotechnical investigation and penetration test, blow count, to obtain bearing capacity of Soils, and allowable stress.

The following is a recommended general scope of work required from a Geotechnical Engineering office:

Preliminary visual identification of soils: See Army Core of Engineers Document Online:
(Recommended to be performed as a minimum on medium and large residential projects)
http://www.usace.army.mil/publications/eng-manuals/em1110-1-1804/appendF-3.pdf
And - Bearing Capacity of Soils:
http://www.usace.army.mil/publications/eng-manuals/em1110-1-1905/toc.htm
Also if interested see List of Documents:
http://www.usace.army.mil/publications/eng-manuals/em.htm

GENERAL SCOPE FOR GEOTECHNICAL INVESTIGATION:

Foundation Design Studies and Recommendations

1. Foundation and Lowest floor slab Recommendations:

    a. Allowable Bearing Pressure of Soil or Rock and Minimum depth of footings
    b. Recommended foundation system
    c. Alternative Foundation systems considered
    d. Lateral Earth Support and Protection of existing structures

3. Information on Design Groundwater level and recommendations on water proofing

4. Earthquake considerations
   Site Coefficient "S"

5. Lateral Earth Pressures

6. Stability of slopes

Construction Considerations

1. Excavation, Dewatering and Protection of subgrade

2. Lateral Earth Support and Protection of existing structures. Temporary excavation and protection such as excavation sheeting underpinning and temporary dewatering systems

3. Backfilling and use of onsite soils as backfill

4. Construction monitoring

Foundation Types for Buildings, shallow footings, deep footings, foundation walls, soil retaining structures, lateral earth pressure.

Introduction to Torsion - Saint-Venant Torsion Twist - and Warping Torsion, Compression due to twist.